

COOPERATIVE SEARCH ADVERTISING

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ABSTRACT

Channel coordination in search advertising is an important but complicated managerial decision for both manufacturers and retailers. A manufacturer can sponsor retailers to advertise his products while at the same time compete with them in position auctions. We model a manufacturer and retailers' cooperation and intra-brand competition in advertising, as well as inter-brand competition with other advertisers. We consider a simple coordination mechanism where a manufacturer shares a fixed percentage of a retailer's advertising cost. Our model prescribes the optimal cooperative advertising strategies from the manufacturer's perspective. We find that it can be optimal for a manufacturer not to cooperate with all his retailers even if they are ex ante the same. This reflects the manufacturer's tradeoff between a higher demand versus a higher bidding cost resulting from more competition. We also find that an advertiser's position rank in equilibrium is entirely determined by his channel profit per click, no matter he is a manufacturer or retailer. Consequently, when determining whether to advertise directly to consumers or via retailers, a manufacturer should compare his profit per click from direct sales with the retailer's total channel profit per click; similarly, when choosing which retailer to sponsor, a manufacturer should compare their total channel profits per click. We also investigate how a manufacturer uses both wholesaling and advertising contracts to coordinate channels with endogenous retail prices.

Keywords: *Search Advertising; Position Auctions; Cooperative Advertising; Channel Coordination*

1. INTRODUCTION

Search advertising is growing rapidly and has become a major advertising channel. In 2015, compared to a 6.0% growth of the entire advertising industry, search advertising grew fast at 16.2%, and has reached a global expenditure of \$80 billion dollars¹. Retailing is a major contributor to search advertising. The number one spender on Google Adwords is Amazon, and five of the top ten industries contributing most to Google Adwords are related to retailing. These industries include retailing and general merchandise, home and garden, computer and consumer electronics, vehicles, as well as business and industrial, which together contribute more than one quarter of Google's revenue.² In all these industries, both brands and retailers can advertise on the same keywords. For example, Figure 1 shows Google ads in one search query of "laptop". In this example, advertisers include nine brands of laptop manufacturers and two retailers. Some manufacturers advertise products via their own e-commerce sites (Microsoft Surface, Samsung and Google Chromebook), some advertise products via retailers (Asus, Apple, Lenovo, Dell, and Toshiba), and some, such as HP, do both. Further explorations on other keywords reveal more heterogeneities in the market structure. There are cases where both brands and retailers advertise; there are also cases where only retailers advertise.

What lies behind the scene is the channel coordinations between manufacturers and retailers on search advertising. On one hand, manufacturers and their retailers compete directly with each other on search engine platforms; on the other hand, it is common for a manufacturer to coordinate with his retailers on search advertising spending, i.e., to engage in the so-called cooperative advertising. Specifically, a manufacturer can set up a "co-op" fund and distribute money to retailers, who then use the money to advertise the manufacturer's products on search engines. As an example where the co-op contract is publicly available, the state government of Connecticut provides a one-to-one match to local hotels for advertising dollars spent on search engines, so as to encourage them to make more advertisements to attract more tourists to visit the state.³

Cooperative advertising is not new, and it prevails in traditional media. For example, brands, such as Procter and Gamble, can promote their products and get them better displayed by subsidizing supermarkets. However, cooperative advertising on search engines has distinct features compared to that in traditional media. In search advertising, the positions of advertisements are determined through position auctions. Therefore, a manufacturer and his retailers compete directly in bidding for a better position. One advertiser's higher bid will either increase other advertisers'

¹Data source: eMarketer.com.

²This is according to a breakdown of Google's 2011 revenue provided by WordStream.com.

³See details at <http://www.ct.gov/cct/cwp/view.asp?a=3933&q=302600>.

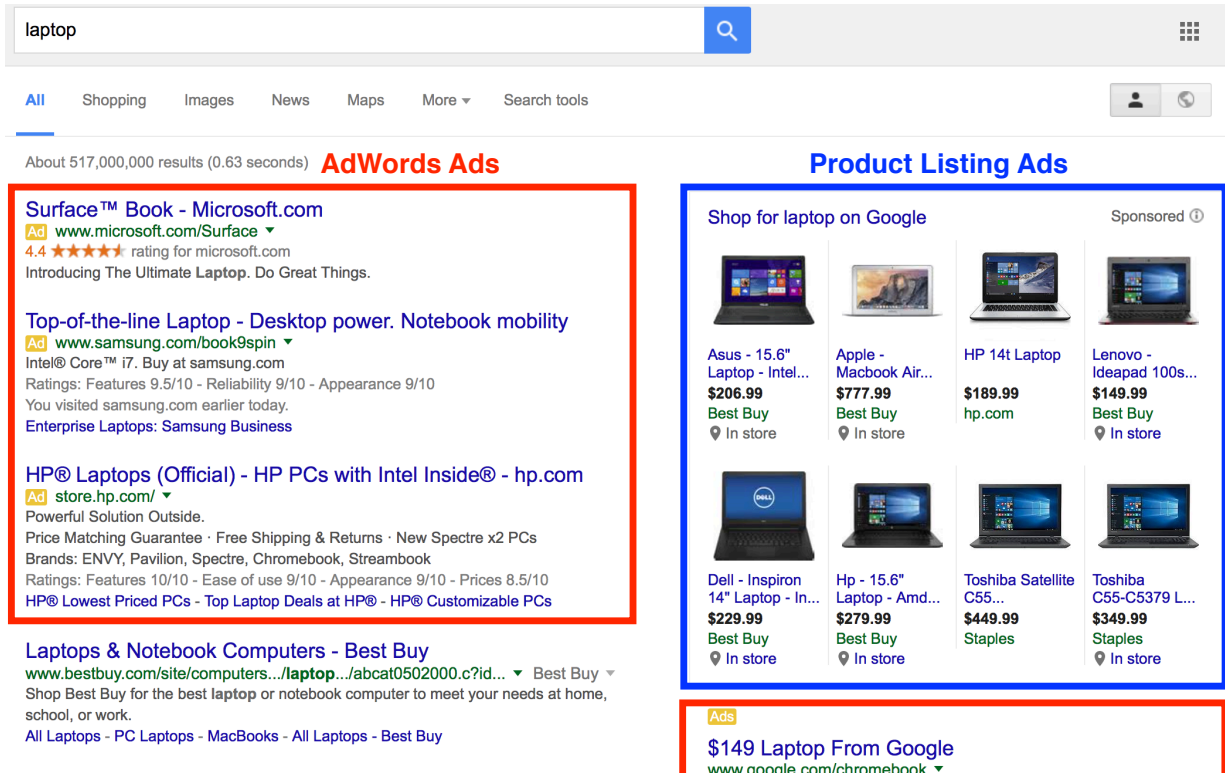


Figure 1: An example of Google ads on “laptop”. Google provides two kinds of search ads: AdWords ads in texts and product listing ads in pictures.

costs or decrease their demands by moving them to less attractive positions. In contrast, in traditional advertising, a manufacturer and his retailers generally do not compete head to head in advertising, and in fact their ads usually have complementary effect on demands.⁴ This new feature in cooperative search advertising has been noticed by brand managers. For example, a quote from a senior manager of search engine and mobile marketing at HP states that “We’re driving each other’s bidding up. HP’s perspective is that we don’t think co-op is that positive if we’re all going after the same term.” Meanwhile, several recent studies and white papers by eMarketer.com and Interactive Advertising Bureau (IAB) have also found that “Digital marketing is changing the cooperative advertising landscape and offering both brands and local businesses new opportunities to reach customers. However, many brands are not only struggling to keep up in this digital era, but also leaving more advertising dollars—about \$14 billion—unused. Half of US brand marketers

⁴In traditional media, a manufacturer and his retailers face weaker competition on advertisements with each other, due to several reasons. First, their ads can differ in contents thus serve different purposes. For example, a manufacturer can make national ads about the introduction of a new product, and retailers can make local ads about the availability, price, promotion, service, etc. Second, in contrast to search advertising where the ads are mainly made to drive conversions, ads in traditional media have more complex effects on demand, such as increasing brand awareness. A consumer who has watched a TV ad from the manufacturer may choose to buy the product from a retailer. Therefore, there can positive spillover effect of ads between a manufacturer and his retailers.

said that a lack of digital marketing understanding was a key obstacle to participating in a coop program.”⁵

In this paper, we build a game-theoretic model to understand how manufacturers and retailers coordinate in search advertising, and prescribe manufacturers’ optimal cooperative search advertising strategies. Specifically, we aim to answer the following research questions. (1) Should a manufacturer cooperate with all his retailers? Is it indeed burning his own money by invoking competition on the search advertising platform? (2) If a manufacturer should not cooperate with all retailers, which one(s) should he choose? (3) Given the profit margin via direct sales is often higher than that via retailers, should a manufacturer advertise directly to consumers or via retailers? (4) How to coordinate the channel by using both wholesale and cooperative advertising contracts?

We first build our main model by assuming exogenous wholesale contracts and retail prices. This is a good assumption if we consider that search advertising is only one channel of online sales. We will consider endogenous wholesale contracts and retail prices in Section 3. We consider a simple coordination mechanism that a manufacturer will cover a fixed percentage—so-called *participation rate*—of a retailer’s spending on search advertising. This coordination mechanism has been widely accepted in industry as well as in previous literature (Bergen and John 1997). In the example aforementioned, by one-to-one match, the Connecticut government indeed provides a participation rate of 50%. We consider the following game: first, a manufacturer determines the participation rate for each retailer, and then retailers, possibly the manufacturer himself, and other advertisers submit their bids to a search engine platform. Following the literature of position auctions (Varian 2007, Edelman et al. 2007), we do not explicitly model consumers’ searching and clicking behaviors; instead, we assume an exogenous click-through rate for each position, irrelevant of advertisers’ identities. We will consider an extension that allows the click-through rate to depend on advertisers’ identities in Section 4. We focus on the intra-brand competition among a manufacturer and retailers, but also account for inter-brand competition by incorporating outside advertisers. We analyze a manufacturer’s cooperative search advertising strategy in equilibrium.

We find that it may not be optimal for a manufacturer to cooperate with all the retailers, even when the retailers are ex ante the same. In determining how many retailers to cooperate with, a manufacturer makes a tradeoff between higher demand brought by more retailers and higher bidding cost associated with intensified competition in bidding. The optimal number of retailers to cooperate with will depend on both the manufacturer’s and retailers’ profit per click. With two retailers, we find that the manufacturer will support the retailer with higher total channel profit

⁵<http://www.emarketer.com/Article/Digital-Media-Key-Co-Op-Program-Growth-Brands-Lag/1012892> and <http://www.iab.com/wp-content/uploads/2015/07/CoopAdvertisingStudy.pdf>.

per click to get a higher position than the other retailer. That is, when deciding the two retailers' relative positions, the manufacturer acts as if the channel is integrated, even though the channel is not fully coordinated. This illustrates the effectiveness of this simple coordination mechanism and thus provides a rationale for its prevalence in industry. Lastly, we also find that when the manufacturer can submit bids directly, he will sponsor a retailer to get a higher position than himself as long as the retailer's total channel profit per click is higher than the manufacturer's profit per click from direct sales. This happens even if the manufacturer can get a higher profit margin via direct sales on his own site.

We consider two extensions of our main model. In Section 3, we consider endogenous wholesale contracts and retail prices. Specifically, we consider both linear contracts and two-part tariffs. Under the linear contracts, our model illustrates how a manufacturer uses the two devices—wholesale and advertising contracts—to coordinate the channel. We find that it can still be optimal for a manufacturer to cooperate with one retailer on advertising, but it is never optimal to support both retailers. Both the wholesale and retail prices decrease and then increase as the outside advertiser gets more competitive in the auction. With two-part tariffs, we show that cooperative advertising is no longer needed. This is consistent with the general observation that a sufficiently flexible wholesale contract will fully coordinate the channel and there will be no need for a manufacturer to cooperate with retailers separately on advertising. In Section 4, we extend our main model by allowing the click-through rate to depend on the advertiser's identity. We find that our main results generalize nicely. With two retailers, the manufacturer will support the retailer with higher total channel profit per *impression* to get a higher position than the other retailer.

Our paper contributes to the literature of cooperative advertising (Berger 1972, Desai 1992, Bergen and John 1997, Kim and Staelin 1999, etc.). For traditional media, it has been assumed that demand responds to advertising as a smooth and concave function, and different parties' advertisements have positive spillovers of demands. In this context, cooperative advertising has been suggested as a manufacturer's instrument to incentivize retailers to advertise more so as to leverage the positive demand externalities. In contrast, in the context of cooperative search advertising, a manufacturer can manipulate retailers' positions and thus demand by carefully designing the participation rates. Cooperative advertising becomes an effective instrument for a manufacturer to control retailers' demand. More generally, this paper contributes to the large literature of channel coordination (e.g., Jeuland and Shugan 1983, McGuire and Staelin 1983, Moorthy 1987, etc.), especially if we view advertising as a non-price instrument (Winter 1993, Iyer 1998).

This paper also contributes to a large literature of competitive strategies in search advertising. Existing theoretical works have studied the impact of click fraud on advertisers' bidding strategies

and the search engine’s revenue (Wilbur and Zhu 2009), the interaction between firms’ advertising auction and price competition (Xu et al. 2011), the interplay between organic and sponsored links (Katona and Sarvary 2010), the bidding strategies of vertically differentiated firms (Jerath et al. 2011), the competitive poaching strategy (Sayedi et al. 2014 and Desai et al. 2014), the impact of advertisers’ budget constraints on their own profits and the platform’s revenue (Lu et al. 2015), etc. However, the problem of channel coordination in search advertising has not been studied.

Lastly, we also contribute to the literature on position auctions. The auction mechanism design and equilibrium properties have been investigated extensively (Edelman et al. 2007, Varian 2007, Feng 2008, Chen and He 2011, Athey and Ellison 2011, Zhu and Wilbur 2011, Dellarocas 2012, etc.), but previous studies all assume that bidders are independent. In our setting, a manufacturer’s profit comes from not only his own position, but also his retailers’ positions, so the bidders are not independent from each other any more. We analyze the equilibrium of position auctions with non-independent bidders.

The paper proceeds as the following. In Section 2, we present our main model. We consider endogenous wholesale contracts and retail prices in Section 3 and identity-dependent click-through rates in Section 4. Lastly, Section 5 concludes the paper.

2. MODEL

2.1. Position Auctions

We will lay out the assumptions on position auctions in this subsection. At the same time, we will also briefly review the equilibrium analysis of positions auctions, closely following Varian (2007).

We consider a generalized second-price (GSP) position auction with two positions and three bidders. The bidder with the i -th highest bid wins the i -th position, and pays the price per click that is equal to the $(i + 1)$ -th highest bid ($i = 1, 2$); the bidder with the lowest bid will not get a position nor clicks, and pays zero.⁶ We consider a pay-per-click mechanism, which has been widely adopted in industry. The click through rate (CTR) of the i -th position is denoted as d_i , which is defined as the fraction of clicks out of all impressions displayed to consumers. To simplify notations, we will view the current position auction equivalently as the one with three positions where the third position has zero CTR, i.e., $d_3 = 0$. It is assumed that a higher position has a higher CTR, i.e., $d_i > d_{i+1}$ for $i = 1, 2, 3$. It is also assumed that the CTR of any given position is

⁶In reality, search engines usually use a modified GSP mechanism, which adjusts the ranking according to advertisers’ quality scores, which, roughly speaking, is the prediction of an advertiser’s CTR. We assume all advertisers have the same quality score for simplicity. In Section 4, we will consider the impact of advertisers’ quality scores.

independent of the identity of the advertiser who takes that position. We will relax this assumption in Section 4. Following Varian (2007) and Edelman et al. (2007), we assume the auction is a complete-information simultaneous game, where each bidder knows others’ valuation, or payoff per click. This assumption can be justified by considering that bidding takes place frequently, and as a result, after many rounds of bidding the bidders will be able to infer the valuations of each other.

It turns out that there are infinite Nash equilibria for the position auction. Varian (2007) and Edelman et al. (2007) have come up with some equilibrium refinement rules. We recap their results with three independent bidders by the following lemma.

LEMMA 1: *Consider three independent bidders competing for two positions, with payoff per click $v_1 \geq v_2 \geq v_3$. In equilibrium, bidder 1 will get the first position, bidder 2 the second, and bidder 3 the third. The equilibrium bids by bidder 2 and 3 are respectively,*

$$b_2^* = \frac{d_1 - d_2}{d_1} v_2 + \frac{d_2}{d_1} v_3, \tag{1}$$

$$b_3^* = v_3. \tag{2}$$

To understand the result, let us first consider bidder 2’s one potential deviation—by bidding higher, he may be able to take the first position. To guard against this deviation, we must have $d_2(v_2 - b_3) \geq d_1(v_2 - b_1)$. Varian (2007) proposed the concept of symmetric Nash equilibrium (SNE) by requiring that $d_2(v_2 - b_3) \geq d_1(v_2 - b_2)$. This is a stronger condition because $b_1 \geq b_2$, hence SNE is a subset of Nash equilibria (NE). A nice property of SNE is that bidders with higher payoff per click will always get a higher position. Varian (2007) further proposed the equilibrium selection criterion LB (short for *lower bound*), and it selects the lower bound from the range of bids that satisfy SNE.⁷ LB rule implies that $d_1(v_2 - b_2) \geq d_2(v_2 - b_3)$. The interpretation of this requirement is that, if it happens that bidder 1 bid so low that bidder 2 slightly exceeded bidder 1’s bid and moved up to the first position, bidder 2 will earn at least as much profit as he makes now at the second position. Together, we have $d_2(v_2 - b_3^*) = d_1(v_2 - b_2^*)$, by which we can get the expression of b_2^* in equation (1). One can verify that $b_3^* = v_3$ is also an SNE and satisfies the LB equilibrium selection rule. Therefore, under Varian (2007)’s SNE and LB equilibrium selection criteria, the first bidder that does not win a position will bid truthfully, and the bidders who get a position will underbid.

⁷Edelman et al. (2007) proposed the concept of “locally envy-free equilibria” which requires that each bidder cannot improve her payoff by exchanging bids with the bidder ranked one position above her, and it yields the same result as the LB of SNE in Varian (2007).

2.2. The Game

In this subsection, we continue to specify the assumptions on the game of channel coordinations. Consider a channel with one manufacturer and n retailers ($n \geq 1$). The manufacturer sells one product, either via the retailers or directly to consumers. The manufacturer first signs a wholesale contract with each retailer. The wholesale contract between the manufacturer and each retailer i essentially determines the manufacturer's and retailer i 's profit margins from each product sold via the retailer, which are denoted as m_i and r_i respectively. When the manufacturer sells the product directly to consumers, his unit profit margin is denoted as m_0 . The value of m_i , r_i , and m_0 are assumed to be exogenously given when the manufacturer and retailers make their search advertising decisions. This is a reasonable assumption, since in reality the manufacturer usually supplies a retailer via both online and offline channels, and even for the online channel, search advertising is only part of the demand source. We will consider endogenous pricing decisions as an extension in Section 3. The conversion rate at retailer i 's site is denoted as θ_i , which means that out of all clicks on the retailer's site, θ_i of them will lead to purchases eventually. The conversion rate at the manufacturer's site is denoted as θ_0 . We assume that the conversion rate is independent of the position and hence the bid of the sponsored link. This assumption is consistent with some recent empirical findings (e.g., Narayanan and Kalyanam 2015). Given these assumptions, the manufacturer's profit per click on his own sponsored link is $\theta_0 m_0$. For retailer i 's sponsored link, retailer i 's profit per click is $\theta_i r_i$, the manufacturer's profit per click is $\theta_i m_i$, and the total channel profit per click is $\theta_i(m_i + r_i)$. Besides the intra-brand competition among the manufacturer and retailers, we also consider inter-brand competition in the auction by introducing outside advertisers representing other brands. If a retailer carries multiple brands, we assume that her bidding decisions across brands are independent. The set of outside advertisers is denoted as \mathcal{A} . For each $a \in \mathcal{A}$, her profit per click is assumed to be v_a . We will specify \mathcal{A} for each model setup below.

We consider the following game. First, the manufacturer decides the participation rate $\alpha_i \in [0, 1)$ for each retailer i , which means that the manufacturer will contribute α_i percentage of retailer i 's spending on search advertising, and retailer i only needs to pay the remaining $1 - \alpha_i$ percentage. Second, each retailer i decides her bid b_i , the manufacturer may also bid b_0 for his own site, and outside advertiser a decides his bid b_a ($a \in \mathcal{A}$). Lastly, given everyone's bid, the auction outcome realize, and advertisers' demands and profits realize.

The following lemma shows how participation rate α_i from the manufacturer changes retailer i 's bidding strategy.

LEMMA 2: *Given the manufacturer's participation rate α_i , retailer i 's equivalent profit per click in the position auction will be $\theta_i r_i / (1 - \alpha_i)$. In other words, her bidding strategy will be the same as if*

her profit per click was $\theta_i r_i / (1 - \alpha_i)$ but had no support from the manufacturer.

Proof. Suppose the retailer will get position i and pay p_i in equilibrium. Then for any $j \neq i$, the Nash equilibrium condition is that

$$d_i[\theta_i r_i - (1 - \alpha_i)p_i] \geq d_j[\theta_i r_i - (1 - \alpha_i)p_j], \quad (3)$$

which is equivalent to

$$d_i[\theta_i r_i / (1 - \alpha_i) - p_i] \geq d_j[\theta_i r_i / (1 - \alpha_i) - p_j]. \quad (4)$$

Similarly we can write down and transform the SNE and LB conditions. Therefore, the retailer's equilibrium bid will be the same as if her profit per click was $\theta_i r_i / (1 - \alpha_i)$ but without support from the manufacturer. ■

Lemma 2 shows that, by choosing participation rate α_i , the manufacturer essentially chooses retailer i 's equivalent profit per click on $[\theta_i r_i, +\infty)$. The manufacturer can incentivize retailer i to bid as high as possible by choosing α_i close to one. Moreover, in the position auction, outside bidders do not need to observe the participation rate α_i nor profit per click $\theta_i r_i$; instead, they only need to know each retailer's equivalent profit per click $\theta_i r_i / (1 - \alpha_i)$, which has been assumed to be observable due to repeated bidding. Therefore, the results of our model will not rely on the observability of the channel coordination contract, which has been shown to be a critical assumption that greatly influences the equilibrium channel structure (Coughlan and Wernerfelt 1989).

We build the model progressively by considering three separate setups below. First we consider the *1R model* where the three bidders consist of one retailer and two outside advertisers. It helps us understand when and to what extent the manufacturer should sponsor a retailer in search advertising. Then we consider the *2R model* where the three bidders consist of two retailers and one outside advertiser. It allows us to investigate whether the manufacturer should sponsor multiple retailers, and which retailer(s) to support. Lastly, we consider the *1M1R model* where the three bidders consist of the manufacturer, one retailer, and one outside advertiser. This case allows us to investigate when the manufacturer should bid by himself and when he should support a retailer get a higher position. In theory, we could have a unified setup by considering a position auction of four bidders including the manufacturer, two retailers, and one outside advertiser. However, this will make the analysis technically more complex and tedious, without gaining much more insights.

2.3. Model 1R

In this subsection, we consider a channel in which the manufacturer has only one retailer ($n = 1$) and there are two outside advertisers ($|\mathcal{A}| = 2$). The profit per click of outside advertisers A and B are denoted as v_A and v_B respectively. Without loss of generality, we assume $v_A \geq v_B$. The manufacturer cannot bid by himself.

We start with the benchmark case where the channel is integrated, so that the manufacturer and the retailer will bid like a single agent with the profit per click as $\theta_1(m_1 + r_1)$. We consider three cases below.

In the first case with $\theta_1(m_1 + r_1) \geq v_A$, in equilibrium, the integrated channel will get position 1, and pay advertiser A's bid $(d_1 - d_2)/d_1 \cdot v_A + d_2/d_1 \cdot v_B$ as given by equation (1), so the integrated channel's profit will be,

$$\Pi_C = d_1 \left[\theta_1(m_1 + r_1) - \left(\frac{d_1 - d_2}{d_1} v_A + \frac{d_2}{d_1} v_B \right) \right] = d_1 [\theta_1(m_1 + r_1) - v_A] + d_2(v_A - v_B). \quad (5)$$

In the second case with $v_A > \theta_1(m_1 + r_1) \geq v_B$, in equilibrium, the integrated channel will get position 2, and pay v_B for each click, so the integrated channel profit's will be,

$$\Pi_C = d_2 [\theta_1(m_1 + r_1) - v_B]. \quad (6)$$

Finally, in the third case with $v_B > \theta_1(m_1 + r_1)$, in equilibrium, the integrated channel will get position 3, so the channel profit will be zero.

To summarize, the integrated channel's profits in the three cases are as follows:

$$\Pi_C = \begin{cases} d_1 [\theta_1(m_1 + r_1) - v_A] + d_2(v_A - v_B), & \theta_1(m_1 + r_1) \geq v_A \text{ (pos 1)}, \\ d_2 [\theta_1(m_1 + r_1) - v_B], & v_A > \theta_1(m_1 + r_1) \geq v_B \text{ (pos 2)}, \\ 0, & v_B > \theta_1(m_1 + r_1) \text{ (pos 3)}, \end{cases} \quad (7)$$

where (pos i) denotes the position that the integrated channel will get in equilibrium.

Now, let us consider the decentralized channel. Given the manufacturer's participation rate α_1 , we know from Lemma 2 that the retailer's *equivalent* profit per click in the position auction will be $\theta_1 r_1 / (1 - \alpha_1)$. We assume that $v_A \geq v_B \geq \theta_1 r_1$, i.e., without support from the manufacturer, the retailer cannot win a position by herself. This is the most interesting case to study because this is the case where different levels of participation rates can move the retailer to different positions. This assumption also corresponds to reality most closely, where the number of outside advertisers

is usually large. Similar to the analysis of the integrated channel above, we consider three cases below, and solve the equilibrium by backward induction.

In the first case, the retailer gets position 1. The manufacturer's profit will be

$$\pi_M(\alpha_1) = d_1 \left[\theta_1 m_1 - \alpha_1 \left(\frac{d_1 - d_2}{d_1} v_A + \frac{d_2}{d_1} v_B \right) \right], \quad (8)$$

which decreases in α_1 . Therefore, the manufacturer will choose the smallest α_1 that satisfies $\theta_1 r_1 / (1 - \alpha_1) \geq v_A$, to guarantee that retailer will indeed get position 1. The optimal choice of α_1 is,

$$\alpha_1^* = 1 - \frac{\theta_1 r_1}{v_A}. \quad (9)$$

Correspondingly, the manufacturer's profit will be,

$$\pi_M(\alpha_1^*) = d_1 \left[\theta_1 m_1 - \alpha_1^* \left(\frac{d_1 - d_2}{d_1} v_A + \frac{d_2}{d_1} v_B \right) \right] = d_1 [\theta_1 (m_1 + r_1) - v_A] + d_2 \frac{v_A - \theta_1 r_1}{v_A} (v_A - v_B), \quad (10)$$

and the retailer's profit will be

$$\pi_R(\alpha_1^*) = d_1 \left[\theta_1 r_1 - (1 - \alpha_1^*) \left(\frac{d_1 - d_2}{d_1} v_A + \frac{d_2}{d_1} v_B \right) \right] = d_2 \frac{\theta_1 r_1}{v_A} (v_A - v_B). \quad (11)$$

The total channel profit will be,

$$\pi_C(\alpha_1^*) = \pi_M(\alpha_1^*) + \pi_R(\alpha_1^*) = d_1 [\theta_1 (m_1 + r_1) - v_A] + d_2 (v_A - v_B). \quad (12)$$

Similarly, we can work out the other two cases when the retailer gets the second position and the third position. We compare the manufacturer's profits among the three cases to determine which one is the equilibrium. To summarize the result, we have the manufacturer's optimal participation rate as

$$\alpha_1^* = \begin{cases} 1 - \frac{\theta_1 r_1}{v_A}, & \theta_1 (m_1 + r_1) \geq v_A + \frac{d_2}{d_1 - d_2} \frac{v_A - v_B}{v_A} \theta_1 r_1 \text{ (pos 1)}, \\ 1 - \frac{\theta_1 r_1}{v_B}, & v_A + \frac{d_2}{d_1 - d_2} \frac{v_A - v_B}{v_A} \theta_1 r_1 > \theta_1 (m_1 + r_1) \geq v_B \text{ (pos 2)}, \\ 0, & v_B > \theta_1 (m_1 + r_1) \text{ (pos 3)}, \end{cases} \quad (13)$$

From the expression of α_1^* , we can see that as the manufacturer's profit per click $\theta_1 m_1$ gets higher, he is more likely to support the retailer to get a higher position; on the other hand, as the retailer's profit per click $\theta_1 r_1$ gets higher, she is more likely to get supported by the manufacturer, but get less support. This is intuitive to understand. Moreover, we also find that as the retailer's profit per click $\theta_1 r_1$ gets higher, the retailer will be less likely to get the top position if and only if $d_1/d_2 + v_B/v_A < 2$. This is a bit counter-intuitive. In fact, we notice that a retailer's equilibrium position depends on

not only her own profit per click but also the manufacturer's support. When the demand in the first position is not very high and the top outside advertiser is very competitive, the manufacturer may prefer the retailer to stay in position 2, when she has a relatively high profit per click, because even though position 1 brings in more demand, it costs the manufacturer more money to help the retailer to get it.

Correspondingly, the total channel profit in equilibrium is,

$$\pi_C(\alpha_1^*) = \begin{cases} d_1 [\theta_1(m_1 + r_1) - v_A] + d_2(v_A - v_B), & \theta_1(m_1 + r_1) \geq v_A + \frac{d_2}{d_1 - d_2} \frac{v_A - v_B}{v_A} \theta_1 r_1 \text{ (pos 1)}. \\ d_2 [\theta_1(m_1 + r_1) - v_B], & v_A + \frac{d_2}{d_1 - d_2} \frac{v_A - v_B}{v_A} \theta_1 r_1 > \theta_1(m_1 + r_1) \geq v_B \text{ (pos 2)} \\ 0, & v_B > \theta_1(m_1 + r_1) \text{ (pos 3)}. \end{cases} \quad (14)$$

The following proposition compares the total channel profit under the integrated case and that under the decentralized case. It has been shown that vertical integration does not necessarily generate a higher profit than vertical separation in the presence of inter-brand competition (McGuire and Staelin 1983, Bonanno and Vickers 1988, Coughlan and Wernerfelt 1989). This is because the outside advertisers will adjust to the channel's structure change. However, we do find that in our 1R model, vertical integration does generate higher profit. The proof to the following proposition is straightforward thus omitted.

PROPOSITION 1: *Consider a position auction participated by two outside advertisers and one retailer who is supported by a manufacturer. Given the retailer's equilibrium position, the total channel profit is the same with that in the case of an integrated channel. However, in general, the manufacturer may under support the retailer than if integrated, and the retailer may under bid than if integrated, so that her equilibrium position may be lower than if integrated, and the equilibrium total channel profit may be lower than if integrated.*

The general intuition behind this result is that in the decentralized case, the manufacturer does not internalize the retailer's profit margin when choosing the participation rate. Specifically, when deciding whether to support the retailer to move from the third to the second position, the manufacturer act as if the channel is integrated, because he can choose the participation rate such that the retailer earns zero profit and the manufacturer collects all the channel profit at the second position. However, when deciding whether to support the retailer to move from the second to the first position, the manufacturer may under support than if integrated, as the retailer gets positive profit at the first position. Essentially, this is because the second highest bidder will underbid than his true profit per click, as shown by equation (1).

2.4. Model 2R

In this subsection, we consider the case that the manufacturer has two retailers ($n = 2$) and there is one outside advertiser ($|\mathcal{A}| = 1$), and the manufacturer does not bid by himself. The outside advertiser A's profit per click is denoted as v_A . Without loss of generality, we assume $\theta_1(m_1 + r_1) \geq \theta_2(m_2 + r_2)$, i.e., the channel profit per click at retailer 1 is higher than that of retailer 2. Similarly as in the 1R model, we assume that $v_A \geq \theta_i r_i$ ($i = 1, 2$). Under this condition, the two retailers are not able to win position 1 without the manufacturer's support, and different levels of participation rates can potentially move each retailer to different positions. This allows us to investigate whether it is optimal for the manufacturer to sponsor one or both retailers, and how much support he should provide to each retailer. We will also consider the case where v_A is between $\theta_1 r_1$ and $\theta_2 r_2$ below as a robustness check.

Similar to the 1R model, we solve the equilibrium of the 2R model by backward induction. Given the manufacturer's participation rates α_1 and α_2 , the position rank of the three bidders is determined by the order of $\theta_1 r_1 / (1 - \alpha_1)$, $\theta_2 r_2 / (1 - \alpha_2)$, and v_A . This results in six possible position configurations. For each position configuration, we can write down the manufacturer's profit function and maximize it with respect to α_1 and α_2 . Lastly we compare the manufacturer's profits under all six position configurations to determine the manufacturer's optimal choice of participation rates α_1^* and α_2^* . The details are relegated to the appendix.

The following theorem answers the questions of which retailer to cooperate with and who will get a higher position in equilibrium, with proof in the appendix.

THEOREM 1: *Consider a position auction participated by an outside advertiser, and two retailers who are supported by a manufacturer. Assume that $v_A \geq \theta_i r_i$ for $i = 1, 2$. In equilibrium, the retailer with higher total channel profit per click will always take a higher position than the other retailer. If it is optimal for the manufacturer to support only one retailer, he will choose to support this retailer.*

The theorem above shows that when deciding which retailer to support, the manufacturer bases on total channel profit per click instead of his own profit per click. Moreover, the relative position rank in equilibrium is determined by the total channel profit per click instead of retailers' own profits per click. Therefore, when deciding the two retailers' *relative* positions, the manufacturer acts as if the channel is integrated. This is true given the fact that the channel is not fully coordinated under the participation rate mechanism. The intuition is that, in cooperative search advertising, the manufacturer needs to consider not only his own profit per click, but also how much he needs to pay for the retailer to get a good position. When the retailer's own profit per click is relatively

high, she already has relatively high willingness-to-pay for the position, and thus the manufacturer can help her get the position with relatively low cost.

The results in Theorem 1 rely on the assumption that $v_A \geq \theta_i r_i$ ($i = 1, 2$). We also consider the case where v_A is between $\theta_1 r_1$ and $\theta_2 r_2$, with details of the analysis relegated in the appendix. We find that given $\theta_i r_i \geq v_A \geq \theta_j r_j$, retailer i will get a higher position when $\theta_i(m_i + r_i) \geq \theta_j(m_j + r_j)$; but retailer j may not get a higher position when $\theta_j(m_j + r_j) > \theta_i(m_i + r_i)$. In other words, in this case, a retailer with higher total channel profit per click may not necessarily get a higher position; yet, a retailer with both higher total channel profit and higher own profit per click will always get a higher position than the other retailer. Therefore compared with Theorem 1, we get somewhat different implications. The reason behind is that given $\theta_i r_i \geq v_A \geq \theta_j r_j$, the manufacturer will never provide positive participation rate to retailer i , thus will have no control of her bidding. He only needs to decide whether and how much to support retailer j . In this sense, we think the case $v_A \geq \theta_i r_i$ ($i = 1, 2$) is a more realistic setting to study a manufacturer's cooperations with two retailers, as the manufacturer will consider supporting both retailers under some circumstances.

Theorem 1 has not said anything about how many retailers to support. To answer this question, it is natural to consider the case where the two retailers are ex ante the same, i.e., $\theta_1 = \theta_2 = \theta$, $m_1 = m_2 = m$, and $r_1 = r_2 = r$. The key tradeoff here is higher demand versus higher bidding cost resulting from intensified competition. Specifically, by supporting more retailers, the manufacturer will get more demand; but at the same time, the bidding costs will go up as retailers now bid higher. Figure 2 characterizes a manufacturer's optimal cooperative search advertising strategy for two symmetric retailers.

Roughly speaking, the manufacturer provides positive participation rates to both retailers when the total channel profit per click is relatively high; he provides support to neither retailers if both his and each retailer's profit per click are relatively low. More interestingly, we find that the manufacturer will provide positive participation rate to only one retailer when his profit per click is relatively high but the retailers' profit per click is relatively low. In this case, retailers need high participation rates to move up to a higher position, but it is too expensive for the manufacturer to support both retailers. For cooperative advertising in traditional media, it has been noticed that advertisements from different retailers usually have positive spillovers (Bergen and John 1997). In contrast, in cooperative search advertising, advertisements from different retailers compete with each other more directly. As a result, in the traditional cooperative advertising, the manufacturer may set up a co-op program and make it open to all his retailers; while in cooperative search advertising, a manufacturer may optimally offer the co-op program only to a subset of retailers, even if the retailers are ex ante the same. When retailers are heterogeneous, we know that the

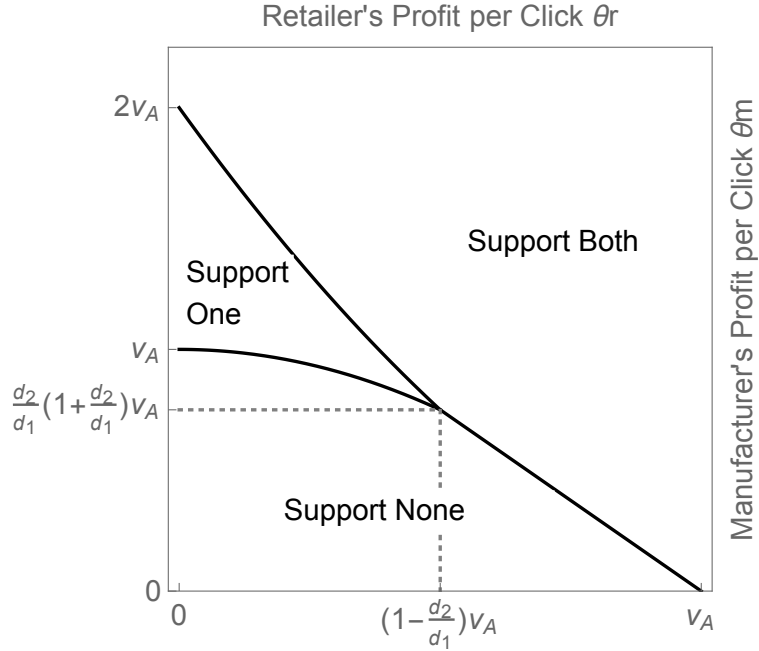


Figure 2: Manufacturer's optimal cooperative search advertising strategy, given two symmetric retailers.

manufacturer would prefer to offer the co-op program to the retailer with higher total channel profit per click according to Theorem 1.

2.5. Model 1M1R

In this subsection, we study the case where the manufacturer has one retailer ($n = 1$) and there is one outside advertiser ($|\mathcal{A}| = 1$), and the manufacturer also submits bid by himself. We assume that $\theta_1 r_1 \leq v_A$ and $\theta_1 r_1 \leq \theta_0 m_0$, i.e., the retailer will end up in position 3 without the manufacturer's support. Again, this is the most interesting case to study as different levels of participation rates from the manufacturer will move the retailer to different positions.

In the position auction, the manufacturer makes profits from both his own site and the retailer's site; therefore, the manufacturer and retailer are non-independent bidders in the auction. As a result, we cannot apply Lemma 1 to calculate the equilibrium bids directly; instead, we consider the following equilibrium concept. For the retailer and the outside advertiser, we still use Varian (2007)'s Symmetric Nash Equilibrium (SNE) and Lower Bound (LB) equilibrium selection rules to determine their equilibrium bids, since they make profits only from their own sites and their choice of bids does not affect their own profits directly at any given position. However, for the manufacturer, his choice of bid does affect his own profit by affecting the price per click on the retailer's site when he is at a lower position than the retailer. As a result, the SNE is no longer a sensible equilibrium

refinement as it does not maximize the manufacturer’s profit with respect to his own bid and thus may fail to be a Nash equilibrium under some circumstances. We will use the manufacturer’s profit maximization (MPM) criterion to replace the SNE and LB in determining his equilibrium bids when he takes a position below the retailer. In fact, MPM is a Nash equilibrium condition when the manufacturer is below and next to the retailer, and it is an equilibrium refinement condition when the manufacturer is below but not next to the retailer. On the other hand, when the manufacturer is above the retailer, his bid does not affect his profit directly, so we still impose the SNE and LB equilibrium selection rules.

Let us now understand the implication of the MPM criterion for the equilibrium analysis when the manufacturer is below the retailer. The SNE and LB equilibrium selection rules for the retailer and outside advertiser will determine their bids as a function of the manufacturer’s bid (like what happens in Lemma 1). Then when the manufacturer follows the MPM rule to pick the most profitable equilibrium, he essentially chooses his bid after taking into account the retailer and outsider advertiser’s responses to his bid. Mathematically, this is as if the manufacturer’s bid is observable to the retailer and outside advertiser. Therefore, the MPM criterion essentially transforms the current simultaneous bidding game equivalently into a dynamic game where the manufacturer first chooses his bid, and then the retailer and outside advertiser choose their bids based on the manufacturer’s choice.

We solve the game by backward induction. In the second stage, given the manufacturer’s participation rate, each bidder’s position will be determined. There are six possible position configurations for the three bidders. In three of the six position configurations, the manufacturer is above the retailer, and in the other three, the manufacturer is below the retailer. For each position configuration with the manufacturer above the retailer, we determine the manufacturer, retailer, and outside advertiser’s bids simultaneously, and then maximize the manufacturer’s profit with respect to α_1 ; for each position configuration with the manufacturer below the retailer, we determine the retailer and outside advertiser’s bids first, and then maximize the manufacturer’s profit with respect to both α_1 and his bid, based on the retailer and outside advertisers’ response functions. Then in the first stage, the manufacturer compare his profits over the six position configurations to determine the equilibrium. The following theorem fully characterizes the equilibrium, with proof in the appendix.

THEOREM 2: *Consider the position auction with one manufacturer, one retailer, and one outside advertiser. Assume that $v_A \geq \theta_1 r_1$ and $\theta_0 m_0 \geq \theta_1 r_1$.*

- *In equilibrium, the manufacturer will take a higher position than the retailer if and only if the manufacturer’s profit per click via direct sales is higher than the retailer’s total channel profit per click, i.e., $\theta_0 m_0 \geq \theta_1 (m_1 + r_1)$.*

- When $\theta_0 m_0 \geq \theta_1(m_1 + r_1)$, the manufacturer provides a positive participation rate to the retailer if and only if $v_A \leq \frac{1}{2}[\theta_1(m_1 + r_1) + \theta_1 r_1]$, and the manufacturer, retailer, and outside advertiser's positions are given by the descending order of $\theta_0 m_0$, $\frac{1}{2}[\theta_1(m_1 + r_1) + \theta_1 r_1]$, and v_A .
- When $\theta_0 m_0 < \theta_1(m_1 + r_1)$, the manufacturer provides a positive participation rate to the retailer if and only if $v_A \leq \theta_1(m_1 + r_1)$, and the manufacturer, retailer, and outside advertiser's positions are given by the descending order of $\frac{1}{2}(\theta_0 m_0 + \theta_1 r_1)$, $\theta_1(m_1 + r_1)$, and v_A .

Similar to our 2R model, we find that the channel member with a higher total channel profit per click will get a higher position in equilibrium. Even if the manufacturer may earn a higher profit margin from direct sales than via the retailer, i.e., $m_0 > m_1$, the manufacturer may still want to sponsor the retailer to get a higher position than himself. There are two reasons for this. First, the conversion rate at the manufacturer's site may be lower than that at the retailer's site, i.e., $\theta_0 < \theta_1$, which may be due to a better design of the retailer's site or consumer loyalty, etc. Second, the retailer's total channel profit may be higher than the manufacturer's profit via direct sales, i.e., $(m_1 + r_1) > m_0$. This can happen due to a higher efficiency of the retailers' operations and logistics, etc. To summarize, the manufacturer will support the retailer to get a higher position in equilibrium if and only if $\theta_1(m_1 + r_1) > \theta_0 m_0$. The intuition is that when the retailer's profit per click $\theta_1 r_1$ is high, the retailer will bid high without the manufacturer's support. Suppose the manufacturer wants to get a high position for his own site, but this requires his bid to exceed the retailer's, which leads to a high bidding cost for the manufacturer. As a result, the manufacturer would rather support the retailer to win a higher position than himself.

Theorem 2 also provides the exact condition under which a manufacturer should support the retailer. When $\theta_0 m_0 < \theta_1(m_1 + r_1)$, the manufacturer takes a lower position than the retailer, and he will support the retailer as long as the outsider advertiser's profit per click is lower than the retailer's total channel profit per click, i.e., $v_A \leq \theta_1(m_1 + r_1)$. On the other hand, when $\theta_0 m_0 \geq \theta_1(m_1 + r_1)$, the manufacturer takes a higher position than the retailer, so a higher bid from the retailer means a higher cost to pay for the manufacturer. As a result, the manufacturer will support the retailer only when the outside advertiser's profit per click is significantly lower than the retailer's total channel profit per click, i.e., $v_A \leq \frac{1}{2}[\theta_1(m_1 + r_1) + \theta_1 r_1] = \theta_1(m_1 + r_1) - \frac{1}{2}\theta_1 m_1 < \theta_1(m_1 + r_1)$.

3. ENDOGENOUS WHOLESALE CONTRACTS AND RETAIL PRICES

In this section, we study the cooperative advertising problem with endogenous wholesale contracts and retail prices. As argued before, the wholesale contracts and retail prices can be seen as ex-

ogenously given when considering that the manufacturer may supply a retailer via both online and offline channels, and even for the online channel, search advertising may be only part of the demand source. However, when this is not the case, it will be more reasonable to consider endogenous wholesale contracts and retail prices, where the advertising, wholesaling, and retailing decisions are coordinated altogether. We will study this problem by considering a setup similar to the 2R model above but now with endogenous wholesale contracts and retail prices.

More specifically, we consider a three-stage game with the timeline of events shown by Figure 3. First, the manufacturer chooses the wholesale contracts and participation rates α_1, α_2 for the two retailers. Second, given the manufacturer’s choices, the two retailers decide retail prices p_1, p_2 . Lastly, after observing the retail prices, the two retailers submit bids b_1, b_2 in a position auction. By specifying the timeline above, we have made several assumptions, and here are our justifications. First, we notice that both wholesale and co-op contracts require heavy administrative work, and thus cannot be altered frequently. In contrast, online retailers can adjust their prices weekly or even daily, so they can treat the wholesale contracts and participations rates as given when determining the retail prices. Moreover, it is not uncommon for retailers to adjust their bids almost continuously with the help of automatic bidding support systems. Therefore it is reasonable to assume that when choosing their bids, the retailers can treat the retail prices as given.

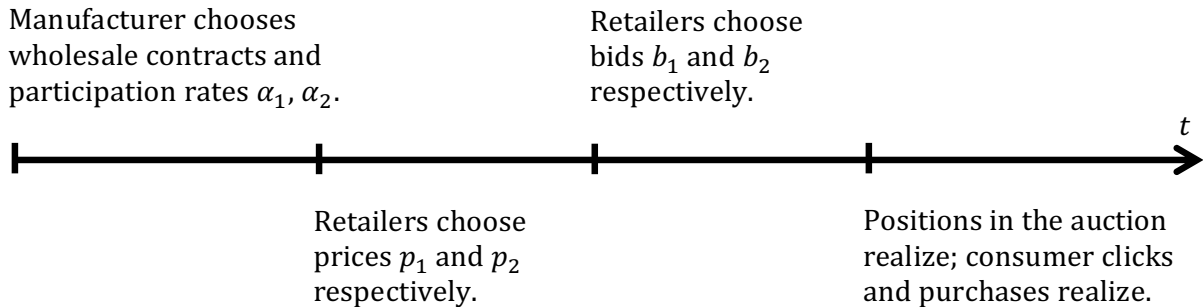


Figure 3: Timeline of events with endogenous wholesale contracts and retail prices.

For the wholesale contracts, we will consider both linear contracts and two-part tariffs below. Generally speaking, two-part tariffs generate a higher channel profit, but linear contracts are simpler to implement and more robust to accommodate against changing environments or incomplete contracts (Villas-Boas 1998). We denote the wholesale prices as w_i for retailer i .

We focus our analysis on text ads (e.g. Google AdWords, as shown in Figure 1). For text ads, retail prices are usually not displayed in the ads, so consumers will not be able to discover the price before clicking an ad. Moreover, we also recognize that there is no monetary cost for consumers to click, and it can be cognitively expensive for consumers to figure out all the back-of-the-envelop

advertising, wholesaling, and retailing coordination decisions. Therefore, we will ignore consumers' possible rational expectations about retail prices before they see them, and instead, assume that the click-through rate of each product ad does not depend on the retail price of the product. However, the price will affect how likely a consumer will make a purchase after they click the ad. That is, retail prices will influence the conversion rate. It is assumed that consumers' valuation of the product is uniformly distributed on $[0, 1]$. Therefore, the conversion rate of retailer i can be written as $\theta_i = \bar{\theta}_i(1 - p_i)$, where $\bar{\theta}_i$ is a constant. Retailer i 's profit margin is $r_i = p_i - w_i$, and the manufacturer's profit margin is $m_i = w_i - c$, where c is the marginal production cost. We will consider ex ante symmetric retailers with $\bar{\theta}_1 = \bar{\theta}_2 = \bar{\theta}$.

We solve the equilibrium by backward induction. In fact, in the last stage when the wholesale contracts, participation rates, and retail prices have been determined, retailers face the exactly same problem as the 2R model above, and we have solved for the retailers' optimal bids given their profit per click. Now, let us consider the two retailers' decisions on retail prices. We first notice that p_i enter into retailer i 's profit function only via $\theta_i r_i$. Retailer i 's equivalent profit per click is,

$$v_i \equiv \frac{\theta_i r_i}{1 - \alpha_i} = \frac{\bar{\theta}(1 - p_i)(p_i - w_i)}{1 - \alpha_i}, \quad (15)$$

which reaches the maximum value $v_i^* = \bar{\theta}(1 - w_i)^2/[4(1 - \alpha_i)]$ when $p_i = (1 + w_i)/2$, and takes the minimum value of zero when p_i is equal to w_i or 1. Therefore, when choosing the retail price $p_i \in [w_i, 1]$, retailer i is essentially choosing $v_i \in [0, v_i^*]$. Given the two retailers' choice of retail prices, their positions will be determined by the order of v_1 , v_2 , and v_A . The following lemma shows that in fact, the bidders' positions in equilibrium are completely determined by the rank of v_1^* , v_2^* , and v_A (with proof in the appendix).

LEMMA 3: *In equilibrium, retail prices are set at $p_i^* = (1 + w_i)/2$, and the positions of retailer 1, retailer 2, and the outside advertiser are given by the descending order of v_1^* , v_2^* , and v_A .*

3.1. Linear Wholesale Contracts

Given the retailers' retail prices and bids, now we consider the manufacturer's problem. Let us first consider linear wholesale contracts. By symmetry, without loss of generality, we can assume that retailer 1 takes a higher position than retailer 2. Similar to the 2R model, we consider three cases depending on which position the outside advertiser takes. Under each case, the three bidders' positions are given, so we can write down the manufacturer's profit function. Then, the manufacturer's optimization problem is to maximize his profit with respect to α_1 , α_2 , w_1 , and w_2 , subject to the constraints that ensure the positions of the three bidders. We relegate the details of calculations in

the appendix. Then, we compare the manufacturer's profit among the three cases, and get his equilibrium wholesale prices and participation rates. The following theorem completely characterizes the equilibrium.

THEOREM 3-A: *Consider a position auction participated by an outside advertiser, and two ex-ante symmetric retailers who are supported by a manufacturer via linear wholesale contracts and participation rates. In equilibrium, the manufacturer's wholesale prices and participations rates are,*

$$\begin{pmatrix} w_1^* \\ w_2^* \\ \alpha_1^* \\ \alpha_2^* \end{pmatrix} = \begin{cases} \left(\frac{1+c}{2}, \frac{1+c}{2}, 0, 0 \right)^T, & v_A \leq \frac{\bar{\theta}(1-c)^2}{16}, \quad (1, 2) \\ \left(1 - 2\sqrt{\frac{v_A}{\bar{\theta}}}, 1 - 2\sqrt{\frac{v_A}{\bar{\theta}}}, 0, 0 \right)^T, & \frac{\bar{\theta}(1-c)^2}{16} < v_A \leq \left(\frac{d_1+d_2+d_2\sqrt{\frac{3d_1+d_2}{d_1+d_2}}}{d_1+3d_2} \right)^2 \frac{\bar{\theta}(1-c)^2}{4}, \quad (1, 2) \\ \left(\frac{d_1c+d_2}{d_1+d_2}, 1, 1 - \frac{\bar{\theta}(1-c)^2d_1^2}{4v_A(d_1+d_2)^2}, 0 \right)^T, & \left(\frac{d_1+d_2+d_2\sqrt{\frac{3d_1+d_2}{d_1+d_2}}}{d_1+3d_2} \right)^2 \frac{\bar{\theta}(1-c)^2}{4} < v_A \leq \frac{2d_1+d_2}{d_1+d_2} \frac{\bar{\theta}(1-c)^2}{8}, \quad (1, 3) \\ \left(\frac{1+c}{2}, 1, 0, 0 \right)^T, & v_A > \frac{2d_1+d_2}{d_1+d_2} \frac{\bar{\theta}(1-c)^2}{8}, \quad (2, 3) \end{cases} \quad (16)$$

where (i, j) at the end of each row indicates that under such condition, retailer 1 and 2 take positions i and j respectively in equilibrium. The retail prices in equilibrium are,

$$p_i^* = \frac{1 + w_i^*}{2}, \quad i = 1, 2. \quad (17)$$

Retailer 1, 2, and outsider advertiser's positions in equilibrium are given by the descending order of $\bar{\theta}(1 - w_1^*)^2/[4(1 - \alpha_1^*)]$, $\bar{\theta}(1 - w_2^*)^2/[4(1 - \alpha_2^*)]$, and v_A .

Basically, the manufacturer has two devices at hands—wholesale prices w_1 and w_2 , and participation rates α_1 and α_2 . Depending on the outside advertiser's profit per click v_A , the manufacturer will optimally apply one or both devices to coordinate the channel so as to maximize his profit. Let us go through the four cases in equation (16) together to understand the manufacturer's optimal wholesaling and advertising strategies. First, when v_A is very low, the two retailers will get the top two positions without the help from the manufacturer. In this case, the manufacturer sets the monopolistic wholesale prices as $(1 + c)/2$, and provides zero participation rates. Now, as we increase v_A to the interval in the second case, the manufacturer still wants to keep the two retailers at the top two positions. He will achieve this goal by providing lower wholesale prices and thus higher profit margins for the retailers but still keeping the participation rates at zero. Intuitively, lowering the wholesale prices not only increases the retailers' profit margins and thus helps the retailers outbid the outside advertiser, but also increases the demand as the retail prices go down; whereas increasing participation rates only has the first effect. This is why the wholesale prices are the manufacturer's first choice when fighting against downstream competition in bidding. Now, as

we further increase v_A to the third case, wholesale prices alone do not suffice to grant the retailers the winners of the auction. The manufacturer will set a low wholesale price and at the same time provide a positive participation rate to retailer 1 so as to keep her at the first position. He will entirely drop retailer 2 by not selling to him, and as a result retailer 2 will take the third position with zero demand. Lastly, when v_A is very high, the manufacturer has to give up winning the auction. He will set the monopolistic wholesale price $(1+c)/2$ again for retailer 1 and provides zero participation rate to her. He will not sell to retailer 2, who will take the third position.

Figure 4 clearly illustrates the manufacturer's optimal wholesaling and cooperative advertising strategies as described above. By Lemma 3, the equilibrium retail price $p_i^* = (1 + w_i^*)/2$. The average retail price for all consumers will be $\bar{p}^* = (d_{(1)}p_1^* + d_{(2)}p_2^*)/(d_{(1)} + d_{(2)})$, where $d_{(i)}$ denotes the CTR for retailer i given her position. According to equation (16), it is straightforward to show that $\bar{p}^* = p_1^* = (1 + w_1^*)/2$. Therefore, the relationship between \bar{p}^* and v_A will be very similar to the relationship between w_1^* and v_A in Figure 4. As the outside advertiser's profit per click increases, the average retail price first decreases then increases.

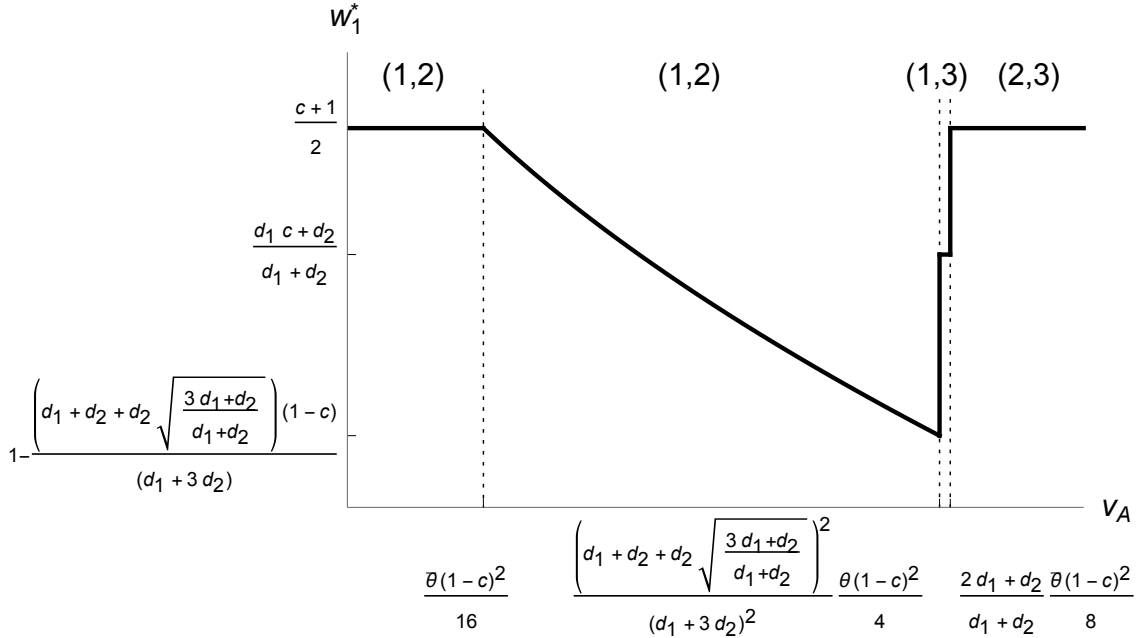


Figure 4: Manufacturer's optimal wholesaling and cooperative advertising strategies under linear wholesale contracts.

3.2. Two-Part Tariffs

In this subsection, we consider the problem where the manufacturer uses two-part tariff wholesale contracts for channel coordination. Each retailer i pays the manufacturer w_i for each product she

sells, as well as a fixed franchise fee. Under two-part tariffs, the retailers' problem is the same as before, with their positions determined by the order of v_1^* , v_2^* and v_A ; however, the manufacturer's objective now is to maximize the total channel profit by choosing w_1 , w_2 , α_1 and α_2 . He uses the franchise fee to divide the channel profit with retailers. Similarly, we analyze the equilibrium given each of the three position configurations in the appendix. By comparing the manufacturer's profits among the three position configurations, we get his equilibrium wholesale prices and participation rates. The following theorem completely characterizes the equilibrium.

THEOREM 3-B: *Consider a position auction participated by an outside advertiser, and two ex-ante symmetric retailers who are supported by a manufacturer via two-part tariffs and participation rates. In equilibrium, the manufacturer's wholesale prices and participations rates are,*

$$\begin{pmatrix} w_1^* \\ w_2^* \\ \alpha_1^* \\ \alpha_2^* \end{pmatrix} = \begin{cases} \left(c, \left(1 - \frac{d_2}{d_1}\right) + \frac{d_2}{d_1}c, 0, 0 \right)^T, & v_A \leq \frac{d_2^2}{4d_1^2} \frac{\bar{\theta}(1-c)^2}{4}, (1, 2) \\ \left(c, 1 - 2\sqrt{\frac{v_A}{\bar{\theta}}}, 0, 0 \right)^T, & \frac{d_2^2}{4d_1^2} \frac{\bar{\theta}(1-c)^2}{4} < v_A \leq \frac{\bar{\theta}(1-c)^2}{9}, (1, 2) \\ \left(c, 1, 0, 0 \right)^T, & \frac{\bar{\theta}(1-c)^2}{9} < v_A \leq \frac{\bar{\theta}(1-c)^2}{4}, (1, 3) \\ \left(c, 1, 0, 0 \right)^T, & v_A > \frac{\bar{\theta}(1-c)^2}{4}, (2, 3) \end{cases}, \text{ if } \frac{d_1}{d_2} \geq \frac{3}{2}. \quad (18)$$

$$\begin{pmatrix} w_1^* \\ w_2^* \\ \alpha_1^* \\ \alpha_2^* \end{pmatrix} = \begin{cases} \left(c, \left(1 - \frac{d_2}{d_1}\right) + \frac{d_2}{d_1}c, 0, 0 \right)^T, & v_A \leq \frac{d_2^2}{4d_1(3d_2-d_1)} \frac{\bar{\theta}(1-c)^2}{4}, (1, 2) \\ \left(c, 1, 0, 0 \right)^T, & \frac{d_2^2}{4d_1(3d_2-d_1)} \frac{\bar{\theta}(1-c)^2}{4} < v_A \leq \frac{\bar{\theta}(1-c)^2}{4}, (1, 3) \\ \left(c, 1, 0, 0 \right)^T, & v_A > \frac{\bar{\theta}(1-c)^2}{4}, (2, 3) \end{cases}, \text{ if } \frac{d_1}{d_2} < \frac{3}{2}. \quad (19)$$

The retail prices in equilibrium are,

$$p_i^* = \frac{1 + w_i^*}{2}, \quad i = 1, 2. \quad (20)$$

Retailer 1, 2, and outsider advertiser's positions in equilibrium are given by the descending order of $\bar{\theta}(1 - w_1^*)^2/[4(1 - \alpha_1^*)]$, $\bar{\theta}(1 - w_2^*)^2/[4(1 - \alpha_2^*)]$, and v_A .

Compared with linear contracts, we find that cooperative advertising in forms of participation rates is never needed to coordinate the channel with two-part tariffs. This is consistent with the general observation that a sufficiently flexible wholesale contract will fully coordinate the channel and there will be no need for a manufacturer to cooperate with retailers separately on advertising. We find that the manufacturer always sets the wholesale price as the marginal production cost for retailer 1, who takes a higher position than the other retailer. This not only eliminates the double marginalization conflicts in retail prices, but also maximizes the retailer's profit per click thus chance of winning the auction. In contrast, it takes the manufacturer more deliberations when setting the wholesale price for retailer 2, who takes a lower position than retailer 1. When the

outside advertiser’s profit per click is relatively high, the manufacturer will not sell to retailer 2, who ends up in the third position; on the other hand, when the outside advertiser’s profit per click is relatively low, the manufacturer will support retailer 2 to take the second position by providing her a wholesale price that is higher than the marginal production cost, in an effort to balance between supporting retailer 2 to outbid the outside advertiser and lowering the price per click for retailer 1.

4. IDENTITY-DEPENDENT CLICK-THROUGH RATE

In our main model above, we have assumed that the CTR at each position is independent of the identity of the advertiser who takes the position. In reality, this assumption may not hold. For example, some consumers may be loyal to a retailer and more likely to click on its sponsored ads even if the retailer is not at the top position. In this section, we consider an extension of our 2R model that allows the CTR of a sponsored ad to depend on both its position and the identity of its advertiser.

Specifically, we assume that the CTR of advertiser i at position j , d_{ij} can be decomposed as $d_{ij} = e_i x_j$, where e_i is the “identity effect”, which measures the attractiveness of the advertiser, and x_j is the “position effect”, which measures the attractiveness of the position.⁸ Similarly as before, it is assumed that given an advertiser, the higher position she takes, the more clicks she will get, i.e., x_j decreases in j . Moreover, it is common for search advertising platforms to adjust the position rank of advertisers according to their identity effects. For example, when deciding the positions ranks, Google augments each advertiser’s bid with her quality score, which, roughly speaking, is a measure of the advertiser’s predicted CTR, i.e., the identity effect, besides other less important considerations such as landing page quality. We follow Varian (2007) to assume that the positions of advertisers are ranked according to $e_i b_i$ in descending order, where b_i denotes the bid of the advertiser at position i . The advertiser at position i then pays $(e_{i+1}/e_i)b_{i+1}$ per click.

The equilibrium analysis of the model here parallels with the 2R model above (with details in the appendix), and summarize the findings by the following theorem.

THEOREM 4: *Consider a position auction with identity-dependent CTR, participated by an outside advertiser, and two retailers who are supported by a manufacturer. Assume that $e_A v_A \geq e_i \theta_i r_i$ for $i = 1, 2$. In equilibrium, the retailer with higher total channel profit per impression will always take a higher position than the other retailer. If it is optimal for the manufacturer to support only one retailer, he will choose to support this retailer.*

⁸We have not fully modeled consumers’ search and click behaviors, which is beyond the scope of the paper. See Athey and Ellison (2011), Chen and He (2011), Jerath et al. (2011), etc.

Theorem 4 generalizes the results in Theorem 1 nicely. With identity-dependent CTR, the retailer with higher total channel profit per impression, i.e., $e_i\theta_i(m_i + r_i)$, will always take a higher position than the other retailer in equilibrium.

5. CONCLUSION

This paper studies cooperative search advertising by considering a simple form of coordination contract—a manufacturer shares a fixed percentage of a retailer’s ads spending. We consider intra-brand competition among one manufacturer and two retailers, as well as inter-brand competition with outside advertisers. We find that it can be optimal not to sponsor both retailers even if they are ex ante the same. This reflects the manufacturer’s tradeoff between a higher demand versus a higher bidding cost resulting from more competition. We also find that an advertiser’s total channel profit per click will determine his position rank in equilibrium, no matter he is a manufacturer or retailer. This illustrates the effectiveness of this simple coordination mechanism despite that it does not fully coordinate the channel.

In general, our main results carry through the two extensions we consider. First, with endogenous linear wholesale contracts and retail prices, the manufacturer may still optimally sponsor a retailer in advertising; however, it is never optimal to sponsor both retailers and it is no longer necessary to use cooperative advertising with two-part tariffs. Second, when click-through rates depend on advertisers’ identities, we show that our main results generalize nicely—now one’s total channel profit per impression will determine his position rank.

There are limitations of the paper. We do not make a distinction between branded and generic keywords, which could be a topic for future research. Also, we restrict ourselves with a simple coordination mechanism in the form of participation rates. It will be interesting to understand the optimal cooperative search advertising contracts. Lastly, we have not fully modeled consumers’ search and click behaviors, and thus cannot evaluate consumers’ welfare in the context of cooperative search advertising.

APPENDIX

EQUILIBRIUM ANALYSIS OF THE 2R MODEL AND PROOF OF THEOREM 1:

There are three possible cases, depending on the position of the outside advertiser. (There are six distinct position configurations.)

- In the first case, the outside advertiser gets position 1. We first assume that the retailer 1 gets position 2 and retailer 2 gets position 3, and we will show that given $\theta_1(m_1 + r_1) \geq \theta_2(m_2 + r_2)$, this is the only possible equilibrium. By equilibrium condition for this position configuration, we have that $v_A \geq \theta_1 r_1 / (1 - \alpha_1) \geq \theta_2 r_2 / (1 - \alpha_2)$.

According to Lemma 1, retailer 2's bid at position 3 will be her equivalent profit per click, $\theta_2 r_2 / (1 - \alpha_2)$, and this is the price per click for retailer 1. The manufacturer's profit will be,

$$\pi_M(\alpha_1, \alpha_2) = d_2 \left(\theta_1 m_1 - \alpha_1 \frac{\theta_2 r_2}{1 - \alpha_2} \right),$$

which decreases in α_1 and α_2 . Therefore, the manufacturer will choose the smallest α_1 and α_2 that satisfy $v_A \geq \theta_1 r_1 / (1 - \alpha_1) \geq \theta_2 r_2 / (1 - \alpha_2)$. The optimal choice will be,

$$\begin{aligned} \alpha_1^* &= \max \left\{ 1 - \frac{\theta_1 r_1}{\theta_2 r_2}, 0 \right\}, \\ \alpha_2^* &= 0. \end{aligned}$$

Correspondingly, the manufacturer's profit under the optimal participation rates will be,

$$\pi_M(\alpha_1^*, \alpha_2^*) = d_2 [\theta_1(m_1 + r_1) - \max\{\theta_1 r_1, \theta_2 r_2\}]. \quad (i)$$

According to the equation above and by symmetry, we can see that given $\theta_1(m_1 + r_1) \geq \theta_2(m_2 + r_2)$, the manufacturer cannot gain higher profit by exchanging the positions of retailer 1 and retailer 2. Therefore, given that the two retailers take the second and third positions, it must be that retailer 1 gets position 2, and retailer 2 gets position 3. Accordingly, profits of the two retailers and the channel under α_1^* and α_2^* are,

$$\begin{aligned} \pi_{R_1}(\alpha_1^*, \alpha_2^*) &= d_2 \left[\theta_1 r_1 - (1 - \alpha_1^*) \frac{\theta_2 r_2}{1 - \alpha_2^*} \right] = \max \{ \theta_1 r_1 - \theta_2 r_2, 0 \}, \\ \pi_{R_2}(\alpha_1^*, \alpha_2^*) &= 0, \\ \pi_C(\alpha_1^*, \alpha_2^*) &= \pi_M(\alpha_1^*, \alpha_2^*) + \pi_{R_1}(\alpha_1^*, \alpha_2^*) + \pi_{R_2}(\alpha_1^*, \alpha_2^*) = d_2 [\theta_1(m_1 + r_1) - \theta_2 r_2]. \end{aligned}$$

To summarize, we have analyzed the case when the outside advertiser gets position 1. We find that the retailer with the higher total channel profit per click will get position 2. The manufacturer needs to provide a positive participation rate to support this retailer to get position 2 only when this retailer's own profit per click is lower than that of the other retailer; otherwise, the manufacturer does not need to provide support to any retailer.

- Now we turn to the second case where the outside advertiser takes position 2. Similarly we assume that retailer 1 gets position 1, and retailer 2 gets position 3, and we will verify that this is the only possible equilibrium in the second case. The equilibrium condition requires that $\theta_1 r_1 / (1 - \alpha_1) \geq v_A \geq \theta_2 r_2 / (1 - \alpha_2)$. According to Lemma 1, retailer 2 will bid her equivalent value $\theta_2 r_2 / (1 - \alpha_2)$, and the outside advertiser will bid $(d_1 - d_2) / d_1 \cdot v_A + d_2 / d_1 \cdot \theta_2 r_2 / (1 - \alpha_2)$. The manufacturer's profit will be,

$$\pi_M(\alpha_1, \alpha_2) = d_1 \left[\theta_1 m_1 - \alpha_1 \left(\frac{d_1 - d_2}{d_1} v_A + \frac{d_2}{d_1} \frac{\theta_2 r_2}{1 - \alpha_2} \right) \right],$$

which decreases in both α_1 and α_2 . Therefore, the manufacturer will choose the smallest α_1 and α_2 that ensure $\theta_1 r_1 / (1 - \alpha_1) \geq v_A \geq \theta_2 r_2 / (1 - \alpha_2)$. The optimal choice will be,

$$\begin{aligned} \alpha_1^* &= 1 - \frac{\theta_1 r_1}{v_A}, \\ \alpha_2^* &= 0. \end{aligned}$$

Correspondingly, the manufacturer's profit under the optimal participation rates will be,

$$\pi_M(\alpha_1^*, \alpha_2^*) = d_1 [\theta_1 (m_1 + r_1) - v_A] + d_2 \frac{(v_A - \theta_1 r_1)(v_A - \theta_2 r_2)}{v_A}. \quad (\text{ii})$$

According to equation above and by symmetry, similarly, we can see that given $\theta_1 (m_1 + r_1) \geq \theta_2 (m_2 + r_2)$, the manufacturer indeed gains higher profit when retailer 1 instead of retailer 2 gets position 1. Accordingly, profits of the two retailers and the channel under α_1^* and α_2^* are,

$$\begin{aligned} \pi_{R_1}(\alpha_1^*, \alpha_2^*) &= d_1 \left[\theta_1 r_1 - (1 - \alpha_1^*) \left(\frac{d_1 - d_2}{d_1} v_A + \frac{d_2}{d_1} \frac{\theta_2 r_2}{1 - \alpha_2^*} \right) \right] = d_2 \theta_1 r_1 \left(1 - \frac{\theta_2 r_2}{v_A} \right), \\ \pi_{R_2}(\alpha_1^*, \alpha_2^*) &= 0, \\ \pi_C(\alpha_1^*, \alpha_2^*) &= d_1 [\theta_1 (m_1 + r_1) - v_A] + d_2 [v_A - \theta_2 r_2]. \end{aligned}$$

To summarize, we have analyzed the case in which the outside advertiser gets position 2. We find that the retailer with the higher total channel profit per click will get position 1. The manufacturer will provide a positive participation rate to this retailer in order to help her get this position. The

manufacturer will not provide support to the other retailer.

• Lastly, we study the third case in which the outside advertiser gets position 3. Similarly, we assume that retailer 1 gets position 1, and retailer 2 gets position 2. We will verify that this is the only possible equilibrium in the third case. The equilibrium condition requires $\theta_1 r_1 / (1 - \alpha_1) \geq \theta_2 r_2 / (1 - \alpha_2) \geq v_A$. According to Lemma 1, the outside advertiser will bid v_A , and retailer 2 will bid $(d_1 - d_2) / d_1 \cdot \theta_2 r_2 / (1 - \alpha_2) + d_2 / d_1 \cdot v_A$. The manufacturer's profit will be,

$$\pi_M(\alpha_1, \alpha_2) = d_1 \left[\theta_1 m_1 - \alpha_1 \left(\frac{d_1 - d_2}{d_1} \frac{\theta_2 r_2}{1 - \alpha_2} + \frac{d_2}{d_1} v_A \right) \right] + d_2 (\theta_2 m_2 - \alpha_2 v_A),$$

which decreases with both α_1 and α_2 . Therefore, the manufacturer will choose the smallest participation rates that ensure $\theta_1 r_1 / (1 - \alpha_1) \geq \theta_2 r_2 / (1 - \alpha_2) \geq v_A$. The optimal choice will be,

$$\begin{aligned} \alpha_1^* &= 1 - \frac{\theta_1 r_1}{v_A}, \\ \alpha_2^* &= 1 - \frac{\theta_2 r_2}{v_A}. \end{aligned}$$

Correspondingly, the manufacturer's profit under the optimal participation rates will be,

$$\pi_M(\alpha_1^*, \alpha_2^*) = d_1 [\theta_1 (m_1 + r_1) - v_A] + d_2 [\theta_2 (m_2 + r_2) - v_A]. \quad (\text{iii})$$

Similarly, we can see that given $\theta_1 (m_1 + r_1) \geq \theta_2 (m_2 + r_2)$, the manufacturer indeed gains higher profit when retailer 1 instead of retailer 2 gets position 1. Therefore, we have again verified that in equilibrium, retailer 1 will get a higher position than retailer 2. Accordingly, profits of the two retailers and the channel under α_1^* and α_2^* are,

$$\begin{aligned} \pi_{R_1}(\alpha_1^*, \alpha_2^*) &= 0, \\ \pi_{R_2}(\alpha_1^*, \alpha_2^*) &= 0, \\ \pi_C(\alpha_1^*, \alpha_2^*) &= d_1 [\theta_1 (m_1 + r_1) - v_A] + d_2 [\theta_2 (m_2 + r_2) - v_A]. \end{aligned}$$

To summarize, we have analyzed the case in which the outside advertiser gets position 3. We find that the retailer with higher channel profit per click will get position 1, and the other retailer will get position 2. The manufacturer will provide positive participation rates to both retailers. Specifically, he can carefully choose the participation rates such that the equivalent profits per click of both retailers are slightly above v_A , which is also the price per click for both positions. As a result, both retailers earn zero profit, and the manufacturer collects the entire channel profit.

So far, we have identified the equilibrium given all possible position configurations of the two

retailers. We find that in equilibrium, retailer 1 always take a higher position than retailer 2, therefore, there are only three possible position configurations in equilibrium. In order to know which position configuration will be chosen by the manufacturer in equilibrium, we only need to compare the manufacturer's profits under the three cases—equations (i), (ii), and (iii). The condition for each position configuration to be the equilibrium is summarized below.

- Retailer 1 and 2 get position 1 and 2 respectively in equilibrium if and only if

$$\theta_2(m_2 + r_2) - v_A > \frac{(v_A - \theta_1 r_1)(v_A - \theta_2 r_2)}{v_A},$$

and $\frac{d_1 - d_2}{d_2} [\theta_1(m_1 + r_1) - v_A] + [\theta_2(m_2 + r_2) - v_A] > v_A - \max\{\theta_1 r_1, \theta_2 r_2\}.$

- Retailer 1 and 2 get position 1 and 3 respectively in equilibrium if and only if

$$\frac{d_1 - d_2}{d_2} [\theta_1(m_1 + r_1) - v_A] > \min\{\theta_1 r_1, \theta_2 r_2\} - \frac{\theta_1 r_1 \cdot \theta_2 r_2}{v_A},$$

and $\theta_2(m_2 + r_2) - v_A < \frac{(v_A - \theta_1 r_1)(v_A - \theta_2 r_2)}{v_A}.$

- Retailer 1 and 2 get position 2 and 3 respectively in equilibrium if and only if

$$\frac{d_1 - d_2}{d_2} [\theta_1(m_1 + r_1) - v_A] < \min\{\theta_1 r_1, \theta_2 r_2\} - \frac{\theta_1 r_1 \cdot \theta_2 r_2}{v_A},$$

and $\frac{d_1 - d_2}{d_2} [\theta_1(m_1 + r_1) - v_A] + [\theta_2(m_2 + r_2) - v_A] < v_A - \max\{\theta_1 r_1, \theta_2 r_2\}.$

ANALYSIS OF THE 2R MODEL WITH v_A BETWEEN $\theta_1 r_1$ AND $\theta_2 r_2$:

In the following analysis, we do not presume that $\theta_1(m_1 + r_1) \geq \theta_2(m_2 + r_2)$. Without loss of generality, suppose $\theta_1 r_1 \geq v_A \geq \theta_2 r_2$. Without any support from the manufacturer, retailer 1 will get position 1, the outside advertiser A will get position 2, and retailer 2 will get position 3. The manufacturer's profit only comes from retailer 1, and it equals to,

$$\pi_M(0, 0) = d_1 \theta_1 m_1.$$

Obviously, the manufacturer has no incentive to give retailer 1 any support, since retailer 1 can already get the first position and thus the manufacturer can get the maximum demand from retailer 1 without paying anything for the clicks. Now let's see whether the manufacturer would like to support retailer 2 to move up.

If the manufacturer moves retailer 2 up to position 2, he needs to provide participation rate α_2 to retailer 2 such that

$$\theta_1 r_1 \geq \frac{\theta_2 r_2}{1 - \alpha_2} \geq v_A. \quad (\text{iv})$$

The outside advertiser's bid at position 3 is v_A , and retailer 2's bid at position 2 is $(d_1 - d_2)/d_1 \cdot \theta_2 r_2 / (1 - \alpha_2) + d_2/d_1 \cdot v_A$, which is the price per click for retailer 1. The manufacturer's profit is then,

$$\pi_M(0, \alpha_2) = d_1 \theta_1 m_1 + d_2 (\theta_2 m_2 - \alpha_2 v_A),$$

which decreases in α_2 , so the manufacturer will choose the smallest α_2 that satisfies (iv), $\alpha_2^* = 1 - \theta_2 r_2 / v_A$. Correspondingly, the manufacturer's profit is

$$\pi_M(0, \alpha_2^*) = d_1 \theta_1 m_1 + d_2 [\theta_2 (m_2 + r_2) - v_A].$$

If the manufacturer further supports retailer 2 to move up to position 1, he needs to provide participation rate α_2 to retailer 2 such that

$$\frac{\theta_2 r_2}{1 - \alpha_2} \geq \theta_1 r_1 \geq v_A. \quad (\text{v})$$

The outside advertiser's bid at position 3 is v_A , and retailer 1's bid at position 2 is $(d_1 - d_2)/d_1 \cdot \theta_1 r_1 + d_2/d_1 \cdot v_A$. Similarly, the manufacturer will choose the smallest α_2 that satisfies (v), so $\alpha_2^* = 1 - \theta_2 r_2 / (\theta_1 r_1)$. The manufacturer's profit is then,

$$\begin{aligned} \pi_M(0, \alpha_2^*) &= d_1 \left[\theta_2 m_2 - \alpha_2 \left(\frac{d_1 - d_2}{d_1} \theta_1 r_1 + \frac{d_2}{d_1} v_A \right) \right] + d_2 \theta_1 m_1 \\ &= d_1 [\theta_2 (m_2 + r_2) - \theta_1 r_1] + d_2 \left[\theta_1 (m_1 + r_1) - \theta_2 r_2 - v_A \left(1 - \frac{\theta_2 r_2}{\theta_1 r_1} \right) \right]. \end{aligned}$$

Comparing the manufacturer's profits in the three scenarios, we can see that the manufacturer will support retailer 2 to get position 2 if and only if,

$$\theta_1 (m_1 + r_1) + \frac{d_2}{d_1 - d_2} \frac{\theta_1 r_1 - v_A}{\theta_1 r_1} \theta_2 r_2 > \theta_2 (m_2 + r_2) > v_A,$$

and the manufacturer will support retailer 2 to get position 1 if and only if,

$$\theta_2 (m_2 + r_2) > \theta_1 (m_1 + r_1) + \frac{d_2}{d_1 - d_2} \frac{\theta_1 r_1 - v_A}{\theta_1 r_1} \theta_2 r_2.$$

To summarize, when v_A is between $\theta_1 r_1$ and $\theta_2 r_2$, the retailer with higher total channel profit does

not necessarily get a higher position. Specifically, given $\theta_1 r_1 \geq v_A \geq \theta_2 r_2$, retailer 1 will get a higher position when $\theta_1(m_1 + r_1) \geq \theta_2(m_2 + r_2)$; but retailer 2 may not get a higher position when $\theta_2(m_2 + r_2) > \theta_1(m_1 + r_1)$. In other words, we need a stricter condition than $\theta_2(m_2 + r_2) > \theta_1(m_1 + r_1)$, as shown above, to grant retailer 2 a higher position than retailer 1.

The reason is that, in the case that $v_A \geq \theta_i r_i$ ($i = 1, 2$), the manufacturer can choose the participation rates such that the bid at position 2 equals to v_A and both retailers pay v_A per click, when the two retailers take the top two positions. Now, consider the case that $\theta_1 r_1 > v_A$. When retailer 1's takes position 2 and retailer 2 takes position 1, retailer 1's bid is a linear combination of v_A and $\theta_1 r_1$, which is greater than v_A . As a result, the price per click at position 1 is higher than v_A . On the other hand, when retailer 1 takes position 1 and retailer 2 takes position 2, the manufacturer can choose retailer 2's participation rate such that the bid at position 2 equals to v_A and both retailers pay v_A per click. Therefore, having retailer 2 at position 1 is more costly and thus demand a stronger condition to ensure it as the equilibrium.

EQUILIBRIUM ANALYSIS OF THE 1M1R MODEL AND PROOF OF THEOREM 2:

We denote the manufacturer's bid as b_M , the retailer's bid as b_R , and the outside advertiser A's bid as b_A . There are six possible position configurations. We use a two-number vector to denote the positions of the manufacturer and the retailer in equilibrium. For example, (3, 1) denotes the position configuration that the manufacturer takes the third position, the retailer takes the first, and correspondingly, the outside advertiser takes the second. We first consider the three cases where the manufacturer takes a higher position than the retailer, i.e., position (2,3), position (1,3), and position (1,2).

- First, let us consider the equilibrium conditions for the case of position (2,3). The position configuration requires that,

$$b_A \geq b_M \geq b_R. \quad (\text{vi})$$

The SNE condition and the LB selection rule that guard against the retailer's deviation from position 3 to position 2 imply that,

$$b_R = \frac{\theta_1 r_1}{1 - \alpha_1}.$$

Since $b_M \geq b_R$, given the equation above, we can show that the SNE condition that guards against the retailer's deviation to position 1 has been satisfied.

The SNE condition and the LB selection rule determine the manufacturer's bid at position 2:

$$b_M = \frac{d_1 - d_2}{d_1} \theta_0 m_0 + \frac{d_2}{d_1} b_R.$$

The SNE condition that guards against the outside advertiser A's deviation from position 1 to position 2 requires that,

$$d_1(v_A - b_M) \geq d_2(v_A - b_R).$$

Given this inequality and $b_M \geq b_R$, the SNE condition that guards against the outside advertiser A's deviation to position 3 must be satisfied.

The manufacturer chooses α_1 to maximize his profit $\pi_M(\alpha_1)$, subject to all the (in)equalities above starting from equation (vi), where,

$$\pi_M(\alpha_1) = d_2(\theta_0 m_0 - b_R) = d_2 \left(\theta_0 m_0 - \frac{\theta_1 r_1}{1 - \alpha_1} \right).$$

The solution to this optimization problem is that,

$$\begin{aligned} \alpha_1^* &= 0, \\ b_R^* &= \theta_1 r_1, \\ b_M^* &= \frac{d_1 - d_2}{d_1} \theta_0 m_0 + \frac{d_2}{d_1} \theta_1 r_1, \\ \pi_M(\alpha_1^*) &= d_2 (\theta_0 m_0 - \theta_1 r_1). \end{aligned}$$

The equilibrium exists if and only if $v_A \geq \theta_0 m_0$.

• Second, let us consider the equilibrium conditions for the case of position (1,3). The position configuration requires that,

$$b_M \geq b_A \geq b_R. \tag{vii}$$

The SNE condition and the LB selection rule that guard against the retailer's deviation from position 3 to position 2 imply that,

$$b_R = \frac{\theta_1 r_1}{1 - \alpha_1}.$$

Since $b_A \geq b_R$, given the equation above, we can show that the SNE condition that guards against the retailer's deviation to position 1 has been satisfied.

The SNE condition and the LB selection rule that guard against the outside advertiser A's deviation from position 2 to position 1 imply that,

$$b_A = \frac{d_1 - d_2}{d_1} v_A + \frac{d_2}{d_1} b_R.$$

Given this equation and $b_A \geq b_R$, the SNE condition that guards against the outside advertiser A's deviation to position 3 has been satisfied.

The manufacturer chooses α_1 to maximize his profit $\pi_M(\alpha_1)$, subject to all the (in)equalities above starting from equation (vii), where,

$$\pi_M(\alpha_1) = d_1(\theta_0 m_0 - b_A) = d_1 \theta_0 m_0 - (d_1 - d_2)v_A - d_2 \frac{\theta_1 r_1}{1 - \alpha_1}.$$

Following the similar procedure, we get the solution to this optimization problem:

$$\begin{aligned} \alpha_1^* &= 0, \\ b_R^* &= \theta_1 r_1, \\ b_A^* &= \frac{d_1 - d_2}{d_1} v_A + \frac{d_2}{d_1} \theta_1 r_1, \\ \pi_M(\alpha_1^*) &= d_1(\theta_0 m_0 - v_A) + d_2(v_A - \theta_1 r_1). \end{aligned}$$

The equilibrium exists for any $v_A \geq \theta_1 r_1$

• Third, let us consider the equilibrium conditions for the case of position (1,2). The position configuration requires that,

$$b_M \geq b_R \geq b_A. \quad (\text{viii})$$

The SNE condition and the LB selection rule that guard against the outside advertiser A's deviation from position 3 to position 2 imply that,

$$b_A = v_A.$$

Given $b_R \geq b_A$ and the equation above, we can show that the SNE condition that guards against the retailer's deviation to position 1 is has been satisfied.

The SNE condition and the LB selection rule that guard against the retailer's deviation from position 2 to position 1 imply that,

$$b_R = \frac{d_1 - d_2}{d_1} \frac{\theta_1 r_1}{1 - \alpha_1} + \frac{d_2}{d_1} b_A.$$

The SNE condition that guards against the retailer's deviation to position 3 is automatically satisfied given $b_R \geq b_A$ and the equation above.

The manufacturer chooses α_1 to maximize his profit $\pi_M(\alpha_1)$, subject to all the (in)equalities above starting from equation (viii), where,

$$\pi_M(\alpha_1) = d_1(\theta_0 m_0 - b_R) + d_2(\theta_1 m_1 - \alpha_1 b_A) = d_1 \theta_0 m_0 - d_2 v_A + d_2 \theta_1 m_1 - (d_1 - d_2) \frac{\theta_1 r_1}{1 - \alpha_1} - \alpha_1 d_2 v_A.$$

Following the similar procedure, we get the solution to this optimization problem:

$$\begin{aligned}\alpha_1^* &= 1 - \frac{\theta_1 r_1}{v_A}, \\ b_A^* &= v_A, \\ b_R^* &= v_A, \\ \pi_M(\alpha_1^*) &= d_1(\theta_0 m_0 - v_A) + d_2[\theta_1(m_1 + r_1) - v_A].\end{aligned}$$

The equilibrium exists for any $v_A \geq \theta_1 r_1$.

Now we consider the other three cases where the manufacturer takes a lower position than the retailer, i.e., position (3,2), position (3,1), and position (2,1).

• First, let us consider the equilibrium conditions for the case of position (3,2). The position configuration requires that,

$$b_A \geq b_R \geq b_M. \quad (\text{ix})$$

Given b_M , the SNE condition and the LB selection rule that guard against the retailer's deviation from position 2 to position 1 imply that,

$$b_R = \frac{d_1 - d_2}{d_1} \frac{\theta_1 r_1}{1 - \alpha_1} + \frac{d_2}{d_1} b_M.$$

Given this equation and $b_R \geq b_M$, the SNE condition that guards against the retailer's deviation to position 3 has been satisfied.

The SNE condition that guards against the outside advertiser A's deviation from position 1 to position 2 requires that,

$$d_1(v_A - b_R) \geq d_2(v_A - b_M).$$

Given this inequality and $b_R \geq b_M$, the SNE condition that guards against the outside advertiser A's deviation to position 3 has been satisfied.

The manufacturer chooses α_1 and b_M to maximize his profit $\pi_M(\alpha_1, b_M)$, subject to all the (in)equalities above starting from equation (ix), where,

$$\pi_M(\alpha_1, b_M) = d_2(\theta_1 m_1 - \alpha_1 b_M).$$

The solution to this optimization problem is that

$$\begin{aligned}\alpha_1^* &= 0, \\ b_M^* &= 0, \\ b_R^* &= \frac{d_1 - d_2}{d_1} \theta_1 r_1, \\ \pi_M(\alpha_1^*) &= d_2 \theta_1 m_1.\end{aligned}$$

The equilibrium exist for any $v_A \geq \theta_1 r_1$.

• Second, let us consider the equilibrium conditions for the case of position (3,1). The position configuration requires that,

$$b_R \geq b_A \geq b_M. \quad (\text{x})$$

Given b_M , the SNE condition and the LB selection rule that guard against the outside advertiser A's deviation from position 2 to position 1 imply that,

$$b_A = \frac{d_1 - d_2}{d_1} v_A + \frac{d_2}{d_1} b_M.$$

Given this equation and $b_A \geq b_M$, the SNE condition that guards against the outside advertiser A's deviation to position 3 has been satisfied.

The SNE condition that guard against the retailer's deviation from position 1 to position 2 imply that,

$$d_1[\theta_1 r_1 - (1 - \alpha_1)b_A] \geq d_2[\theta_1 r_1 - (1 - \alpha_1)b_M].$$

Given the formula of b_A and the inequality above, the SNE condition that guards against the retailer's deviation to position 3 has been satisfied.

The manufacturer chooses α_1 and b_M to maximize his profit $\pi_M(\alpha_1, b_M)$ subject to all the (in)equalities above starting from equation (x), where

$$\pi_M(\alpha_1, b_M) = d_1(\theta_1 m_1 - \alpha_1 b_A).$$

Following the similar procedure, we get the solution to this optimization problem:

$$\begin{aligned}
\alpha_1^* &= 1 - \frac{\theta_1 r_1}{v_A}, \\
b_M^* &= 0, \\
b_A^* &= \frac{d_1 - d_2}{d_1} v_A, \\
\pi_M(\alpha_1^*, b_M^*) &= d_1[\theta_1(m_1 + r_1) - v_A] + d_2(v_A - \theta_1 r_1).
\end{aligned}$$

The equilibrium exist for any $v_A \geq \theta_1 r_1$.

• Lastly, let us consider the equilibrium conditions for the case of position (2,1). The position configuration requires that,

$$b_R \geq b_M \geq b_A. \quad (\text{xi})$$

The SNE condition and the LB selection rule that guard against the outside advertiser A's deviation from position 3 to position 2 imply that,

$$b_A = v_A.$$

Given $b_M \geq b_A$ and the equation above, we can show that the SNE condition that guards against advertiser A's deviation to position 1 is has been satisfied.

Given b_A and b_M , the SNE condition that guards against the retailer's deviation from position 1 to position 2 imply that,

$$d_1[\theta_1 r_1 - (1 - \alpha_1)b_M] \geq d_2[\theta_1 r_1 - (1 - \alpha_1)b_A].$$

Given $b_M \geq b_A$ and the inequality above, the SNE condition that guards against the retailer's deviation to position 3 must be satisfied.

The manufacturer chooses α_1 and b_M to maximize his profit $\pi_M(\alpha_1, b_M)$, subject to all the (in)equalities above starting from equation (xi), where,

$$\pi_M(\alpha_1, b_M) = d_1(\theta_1 m_1 - \alpha_1 b_M) + d_2(\theta_0 m_0 - b_A).$$

The solution to this optimization problem is that,

$$\begin{aligned}\alpha_1^* &= 1 - \frac{\theta_1 r_1}{v_A}, \\ b_A^* &= v_A, \\ b_M^* &= v_A, \\ \pi_M(\alpha_1^*, b_M^*) &= d_1[\theta_1(m_1 + r_1) - v_A] + d_2(\theta_0 m_0 - v_A).\end{aligned}$$

The equilibrium exists for any $v_A \geq \theta_1 r_1$.

So far, we have completely characterizes the equilibrium bids given each position configuration. We only need to compare the manufacturer's profit under each position configuration to determine the equilibrium.

PROOF OF LEMMA 3:

Suppose $v_1^* > v_2^*$, we first prove that retailer 1 will have a higher position than retailer 2 in equilibrium. We prove by contradiction. Suppose in equilibrium retailer 1 has a lower position than retailer 2, then we must have $v_1 < v_2$. If retailer 1 is in position 3, she can deviate by setting v_1 equal to v_1^* , and earn positive profit at position 2 or 1. Thus, it must be that retailer 1 takes position 2, and retailer 2 takes position 1. We have $v_1^* > v_2^* \geq v_A$. In this case, retailer 1's profit $\pi_{R_1} = d_2[v_1 - (1 - \alpha_1)v_A]$, which increases with v_1 , so retailer 1 will choose the largest possible v_1 given her position. We have $v_1 = v_2^*$, and $\pi_{R_1} = d_2[v_2^* - (1 - \alpha_1)v_A]$. Now, if retailer 1 deviates by setting $v_1 = v_1^*$, she will take the first position, and retailer 2 will take the second position. Similarly, given her position, retailer 2 will set $v_2 = v_2^*$ to maximize her profit. As a result, retailer 1 will earn,

$$\pi'_{R_1} = d_1 \left[v_1^* - (1 - \alpha_1) \left(\frac{d_1 - d_2}{d_1} v_2^* + \frac{d_2}{d_1} v_A \right) \right] > d_1 \left[v_1 - (1 - \alpha_1) \left(\frac{d_1 - d_2}{d_1} v_A + \frac{d_2}{d_1} v_A \right) \right] = \pi_{R_1}.$$

This implies that it is not an equilibrium for retailer 1 to take the second position while retailer 2 is taking the first position. To summarize, we have proved that in equilibrium, when $v_1^* > v_2^*$, retailer 1 has a higher position than retailer 2.

Next, suppose $v_1^* > v_2^*$, and we need to prove that (i) when $v_A > v_1^*$, the outside advertiser will take the first position; (ii) when $v_1^* > v_A > v_2^*$, the outside advertiser will take the second position; and (iii) when $v_2^* > v_A$, the outside advertiser will take the third position.

(i) is obvious: given $v_A > v_1^* \geq v_1$, the outsider advertiser must take a higher position than retailer 1.

(ii): When $v_1^* > v_A > v_2^*$, we have $v_A > v_2^* \geq v_2$, so the outside advertiser will always take a higher position than retailer 2. Suppose she takes position 1, then we must have $v_A > v_1$. In this case, retailer 1's profit is $\pi_{R_1} = d_2 [v_1 - (1 - \alpha_1)v_2]$. Similarly as above, we can show that retailer 1 can earn a higher profit by setting v_1 as v_1^* and take the first position instead. Then (ii) is proved.

(iii) is straightforward to prove by contradiction. Suppose $v_2^* > v_A$ and the outside advertiser takes the second position. In this case, retailer 2 will take position 3 and earn zero profit, and she can deviate by setting v_2 at v_2^* and taking the second position instead, which will give her positive profit.

Given that the positions are determined by the rank of v_1^* , v_2^* , and v_A , we know that each retailer i 's profit function increases with v_i , so in equilibrium, retailer i will set retail price $p_i^* = (1 + w_i)/2$, under which v_i takes the maximum value v_i^* . ■

EQUILIBRIUM ANALYSIS OF THE MODEL WITH LINEAR CONTRACTS:

There are three cases depending on the position of the outside advertiser. For each case, we will formulate and then solve the manufacturer's profit maximization problem.

- In the first case, the outside advertiser takes the first position. The manufacturer's optimization problem is,

$$\begin{aligned} & \max_{\alpha_i, w_i} d_2 \left[\bar{\theta} \frac{1 - w_1}{2} (w_1 - c) - \alpha_1 \frac{\bar{\theta}(1 - w_2)^2}{4(1 - \alpha_2)} \right] \\ \text{s.t.} \quad & v_A \geq \frac{\bar{\theta}(1 - w_1)^2}{4(1 - \alpha_1)} \geq \frac{\bar{\theta}(1 - w_2)^2}{4(1 - \alpha_2)}, \\ & 0 \leq \alpha_i < 1, 0 \leq w_i \leq 1, \text{ for } i = 1, 2. \end{aligned}$$

The optimal solution is,

$$\begin{cases} w_1^* = \frac{c+1}{2}, w_2^* = 1, \alpha_1^* = 0, \alpha_2^* \in [0, 1), & v_A \geq \frac{\bar{\theta}(1-c)^2}{16} \\ w_1^* = 1 - 2\sqrt{\frac{v_A}{\bar{\theta}}}, w_2^* = 1, \alpha_1^* = 0, \alpha_2^* \in [0, 1), & v_A \leq \frac{\bar{\theta}(1-c)^2}{16} \end{cases}.$$

Correspondingly, the manufacturer's profit is,

$$\pi_M^* = \begin{cases} d_2 \frac{\bar{\theta}(1-c)^2}{8}, & v_A \geq \frac{\bar{\theta}(1-c)^2}{16} \\ d_2 \left[(1 - c) \sqrt{v_A \bar{\theta}} - 2v_A \right], & v_A \leq \frac{\bar{\theta}(1-c)^2}{16} \end{cases}.$$

To summarize, in this case, the manufacturer essentially only sells to retailer 1, and provides a zero participation rate. The manufacturer sets the monopolistic wholesale price when the outside adver-

tiser's profit per click is relatively high; otherwise, he increases the wholesale price thus decreases the retailer's profit margin and her incentive to bid when the outside advertiser's profit per click is relatively low.

- In the second case, the outside advertiser takes the second position. The manufacturer's optimization problem is,

$$\begin{aligned} & \max_{\alpha_i, w_i} d_1 \left[\bar{\theta} \frac{1-w_1}{2} (w_1 - c) - \alpha_1 \left(\frac{d_1 - d_2}{d_1} v_A + \frac{d_2 \bar{\theta} (1-w_2)^2}{d_1 4(1-\alpha_2)} \right) \right] \\ \text{s.t.} \quad & \frac{\bar{\theta}(1-w_1)^2}{4(1-\alpha_1)} \geq v_A \geq \frac{\bar{\theta}(1-w_2)^2}{4(1-\alpha_2)}, \\ & 0 \leq \alpha_i < 1, 0 \leq w_i \leq 1, \text{ for } i = 1, 2. \end{aligned}$$

The optimal solution is,

$$\begin{cases} w_1^* = \frac{d_1 c + d_2}{d_1 + d_2}, w_2^* = 1, \alpha_1^* = 1 - \frac{\bar{\theta}(1-c)^2 d_1^2}{4(d_1 + d_2)^2 v_A}, \alpha_2^* \in [0, 1), & v_A \geq \frac{\bar{\theta}(1-c)^2 d_1^2}{4(d_1 + d_2)^2} \\ w_1^* = 1 - \sqrt{\frac{4v_A}{\bar{\theta}}}, w_2^* = 1, \alpha_1^* = 0, \alpha_2^* \in [0, 1), & \frac{\bar{\theta}(1-c)^2}{16} \leq v_A \leq \frac{\bar{\theta}(1-c)^2 d_1^2}{4(d_1 + d_2)^2} \\ w_1^* = \frac{1+c}{2}, w_2^* = 1, \alpha_1^* = 0, \alpha_2^* \in [0, 1), & v_A \leq \frac{\bar{\theta}(1-c)^2}{16} \end{cases}$$

Correspondingly, the manufacturer's profit is,

$$\pi_M^* = \begin{cases} \frac{\bar{\theta}(1-c)^2 d_1^2}{4(d_1 + d_2)} - (d_1 - d_2) v_A & \text{when } v_A \geq \frac{\bar{\theta}(1-c)^2 d_1^2}{4(d_1 + d_2)^2} \\ d_1 \left[(1-c) \sqrt{v_A \bar{\theta}} - 2v_A \right] & \text{when } \frac{\bar{\theta}(1-c)^2}{16} \leq v_A \leq \frac{\bar{\theta}(1-c)^2 d_1^2}{4(d_1 + d_2)^2} \\ d_1 \frac{\bar{\theta}(1-c)^2}{8} & \text{when } v_A \leq \frac{\bar{\theta}(1-c)^2}{16} \end{cases}$$

To summarize, in this case, the manufacturer essentially only sells to retailer 1. When the outside advertiser's profit per click is relatively low, the manufacturer sets the monopolistic wholesale price and provides zero participation rate to the retailer at the same time. When the outside advertiser's profit per click is medium, the manufacturer lowers the wholesale price but still provides zero participation rate. A lower wholesale price leaves more profit margin to the retailer thus incentivizes her to bid higher so as to keep the first position. When the outside advertiser's profit per click is relatively high, the manufacturer will set a low wholesale price and provide positive participation rate at the same time so as to keep the retailer in the first position.

- In the third case, the outside advertiser takes the third position. The manufacturer's opti-

mization problem is,

$$\begin{aligned}
& \max_{\alpha_i, w_i} d_1 \left[\bar{\theta} \frac{1-w_1}{2} (w_1 - c) - \alpha_1 \left(\frac{d_1 - d_2 \bar{\theta} (1-w_2)^2}{d_1 4(1-\alpha_2)} + \frac{d_2}{d_1} v_A \right) \right] + d_2 \left[\bar{\theta} \frac{1-w_2}{2} (w_2 - c) - \alpha_2 v_A \right] \\
\text{s.t.} \quad & \frac{\bar{\theta} (1-w_1)^2}{4(1-\alpha_1)} \geq \frac{\bar{\theta} (1-w_2)^2}{4(1-\alpha_2)} \geq v_A, \\
& 0 \leq \alpha_i < 1, 0 \leq w_i \leq 1, \text{ for } i = 1, 2.
\end{aligned}$$

The optimal solution is,

$$\begin{cases}
w_1^* = w_2^* = c, \alpha_1^* = \alpha_2^* = 1 - \frac{\bar{\theta}(1-c)^2}{4v_A}, & \text{when } v_A \geq \frac{\bar{\theta}(1-c)^2}{4} \\
w_1^* = w_2^* = 1 - 2\sqrt{\frac{v_A}{\bar{\theta}}}, \alpha_1^* = \alpha_2^* = 0, & \text{when } \frac{\bar{\theta}(1-c)^2}{16} \leq v_A \leq \frac{\bar{\theta}(1-c)^2}{4} \\
w_1^* = w_2^* = \frac{1+c}{2}, \alpha_1^* = \alpha_2^* = 0, & \text{when } v_A \leq \frac{\bar{\theta}(1-c)^2}{16}
\end{cases}$$

Correspondingly, the manufacturer's profit is,

$$\pi_M^* = \begin{cases}
(d_1 + d_2) \left[\frac{\bar{\theta}(1-c)^2}{4} - v_A \right] & \text{when } v_A \geq \frac{\bar{\theta}(1-c)^2}{4} \\
(d_1 + d_2) \left[(1-c) \sqrt{v_A \bar{\theta}} - 2v_A \right] & \text{when } \frac{\bar{\theta}(1-c)^2}{16} \leq v_A \leq \frac{\bar{\theta}(1-c)^2}{4} \\
(d_1 + d_2) \frac{\bar{\theta}(1-c)^2}{8} & \text{when } v_A \leq \frac{\bar{\theta}(1-c)^2}{16}
\end{cases}$$

To summarize, in this case, the manufacturer sells to both retailers. The wholesale prices and participation rates for the two retailers are ‘‘symmetric’’, i.e., the manufacturer will only provide a marginally lower wholesale price or a marginally higher participation rate to retailer 1 in order to let her get a higher position than retailer 2. When the outside advertiser's profit per click is low, the manufacturer sets the monopolistic wholesale prices and provides zero participation rates to both retailers. When the outside advertiser's profit per click is medium, the manufacturer lowers wholesale prices but still provides zero participation rates. When the outside advertiser's profit per click is relatively high, the manufacturer only charges marginal production cost as wholesale prices, and provides positive participation rates to both retailers, so as to keep them in the first two positions.

EQUILIBRIUM ANALYSIS OF THE MODEL WITH TWO-PART TARIFFS:

The equilibrium analysis here parallels with that for the linear contracts above. There are three cases depending on the position of the outside advertiser. For each case, we will formulate and then solve the manufacturer's channel profit maximization problem.

- In the first case, the outside advertiser takes the first position. The manufacturer's optimiza-

tion problem is,

$$\begin{aligned} & \max_{\alpha_i, w_i} d_2 \left[\bar{\theta} \frac{1 - w_1}{2} \left(\frac{1 + w_1}{2} - c \right) - \frac{\bar{\theta}(1 - w_2)^2}{4(1 - \alpha_2)} \right] \\ \text{s.t.} \quad & v_A \geq \frac{\bar{\theta}(1 - w_1)^2}{4(1 - \alpha_1)} \geq \frac{\bar{\theta}(1 - w_2)^2}{4(1 - \alpha_2)}, \\ & 0 \leq \alpha_i < 1, 0 \leq w_i \leq 1, \text{ for } i = 1, 2. \end{aligned}$$

The optimal solution is,

$$\begin{cases} w_1^* = c, w_2^* = 1, \alpha_1^* = 0, \alpha_2^* \in [0, 1), & v_A \geq \frac{\bar{\theta}(1-c)^2}{4} \\ w_1^* = 1 - 2\sqrt{\frac{v_A}{\bar{\theta}}}, w_2^* = 1, \alpha_1^* = 0, \alpha_2^* \in [0, 1), & v_A < \frac{\bar{\theta}(1-c)^2}{4} \end{cases}.$$

Correspondingly, the channel profit is,

$$\pi_C^* = \begin{cases} d_2 \frac{\bar{\theta}(1-c)^2}{4}, & v_A \geq \frac{\bar{\theta}(1-c)^2}{4} \\ d_2 \left[(1 - c) \sqrt{v_A \bar{\theta}} - v_A \right], & v_A < \frac{\bar{\theta}(1-c)^2}{4} \end{cases}.$$

To summarize, in this case, the manufacturer essentially only sells to retailer 1, and provides a zero participation rate. The manufacturer sets the wholesale price as the marginal production cost when the outside advertiser's profit per click is relatively high; otherwise, he increases the wholesale price thus decreases the retailer's profit margin and her incentive to bid when the outside advertiser's profit per click is relatively low.

- In the second case, the outside advertiser takes the second position. The manufacturer's optimization problem is,

$$\begin{aligned} & \max_{\alpha_i, w_i} d_1 \left[\bar{\theta} \frac{1 - w_1}{2} \left(\frac{1 + w_1}{2} - c \right) - \left(\frac{d_1 - d_2}{d_1} v_A + \frac{d_2}{d_1} \frac{\bar{\theta}(1 - w_2)^2}{4(1 - \alpha_2)} \right) \right] \\ \text{s.t.} \quad & \frac{\bar{\theta}(1 - w_1)^2}{4(1 - \alpha_1)} \geq v_A \geq \frac{\bar{\theta}(1 - w_2)^2}{4(1 - \alpha_2)}, \\ & 0 \leq \alpha_i < 1, 0 \leq w_i \leq 1, \text{ for } i = 1, 2. \end{aligned}$$

The optimal solution is,

$$\begin{cases} w_1^* = c, w_2^* = 1, \alpha_1^* \in [1 - \frac{\bar{\theta}(1-c)^2}{4v_A}, 1), \alpha_2^* \in [0, 1), & v_A > \frac{\bar{\theta}(1-c)^2}{4} \\ w_1^* = c, w_2^* = 1, \alpha_1^* \in [0, 1), \alpha_2^* \in [0, 1), & v_A \leq \frac{\bar{\theta}(1-c)^2}{4} \end{cases}.$$

Under both cases, the channel profit is,

$$\pi_C^* = d_1 \frac{\bar{\theta}(1-c)^2}{4} - (d_1 - d_2)v_A.$$

To summarize, in this case, the manufacturer essentially only sells to retailer 1. He sets the wholesale price as the marginal production cost, and provides the participation rate high enough to help retailer 1 outbid the outside advertiser. The participate rate does not influence the channel profit.

• In the third case, the outside advertiser takes the third position. The manufacturer's optimization problem is,

$$\begin{aligned} \max_{\alpha_i, w_i} & d_1 \left[\bar{\theta} \frac{1-w_1}{2} \left(\frac{1+w_1}{2} - c \right) - \left(\frac{d_1 - d_2}{d_1} \frac{\bar{\theta}(1-w_2)^2}{4(1-\alpha_2)} + \frac{d_2}{d_1} v_A \right) \right] \\ & + d_2 \left[\bar{\theta} \frac{1-w_2}{2} \left(\frac{1+w_2}{2} - c \right) - v_A \right] \\ \text{s.t.} & \quad \frac{\bar{\theta}(1-w_1)^2}{4(1-\alpha_1)} \geq \frac{\bar{\theta}(1-w_2)^2}{4(1-\alpha_2)} \geq v_A, \\ & \quad 0 \leq \alpha_i < 1, 0 \leq w_i \leq 1, \text{ for } i = 1, 2. \end{aligned}$$

The optimal solution is,

$$\begin{cases} w_1^* = c, w_2^* = 1 - \frac{d_2}{d_1}(1-c), \alpha_1^* \in [0, 1), \alpha_2^* = 0, & v_A \leq \frac{d_2^2}{d_1^2} \frac{\bar{\theta}(1-c)^2}{4} \\ w_1^* = c, w_2^* = 1 - 2\sqrt{\frac{v_A}{\bar{\theta}}}, \alpha_1^* \in [0, 1), \alpha_2^* = 0, & \frac{d_2^2}{d_1^2} \frac{\bar{\theta}(1-c)^2}{4} < v_A \leq \frac{\bar{\theta}(1-c)^2}{4} . \\ w_1^* = c, w_2^* = c, \alpha_1^* \in [1 - \frac{\bar{\theta}(1-c)^2}{4v_A}, 1), \alpha_2^* = 1 - \frac{\bar{\theta}(1-c)^2}{4v_A}, & v_A > \frac{\bar{\theta}(1-c)^2}{4} . \end{cases}$$

Correspondingly, the channel profit is,

$$\pi_C^* = \begin{cases} \frac{d_1^2 + d_2^2}{d_1} \frac{\bar{\theta}(1-c)^2}{4} - 2d_2v_A, & v_A \leq \frac{d_2^2}{d_1^2} \frac{\bar{\theta}(1-c)^2}{4} \\ d_1 \frac{\bar{\theta}(1-c)^2}{4} + d_2(1-c)\sqrt{v_A\bar{\theta}} - (d_1 + 2d_2)v_A, & \frac{d_2^2}{d_1^2} \frac{\bar{\theta}(1-c)^2}{4} < v_A \leq \frac{\bar{\theta}(1-c)^2}{4} . \\ (d_1 + d_2) \frac{\bar{\theta}(1-c)^2}{4} - (d_1 + d_2)v_A, & v_A > \frac{\bar{\theta}(1-c)^2}{4} . \end{cases}$$

To summarize, in this case, the manufacturer sells to both retailers. He sets the wholesale price as the production cost and provides high enough participation rate for retailer 1 to ensure she gets the first position. When v_A is relatively low, he sets the wholesale price higher than marginal production cost and provides zero participation rate for retailer 2; when v_A is relatively high, he sets the wholesale price at the marginal production cost and provides positive participation rate for retailer 2.

EQUILIBRIUM ANALYSIS OF THE MODEL WITH IDENTITY-DEPENDENT CTR AND PROOF OF THEOREM 4:

As a counterpart to Lemma 1, Varian (2007) has shown that in equilibrium, $e_1v_1 \geq e_2v_2 \geq e_3v_3$, where v_i denotes the profit per click for the advertiser at position i . The equilibrium bids are,

$$\begin{aligned} b_3 &= v_3, \\ b_2 &= \frac{x_1 - x_2}{x_1}v_2 + \frac{x_2}{x_1} \frac{e_3}{e_2}v_3. \end{aligned}$$

Similarly with the 2R model, we assume that $e_Av_A > e_1\theta_1r_1, e_2\theta_2r_2$. We analyze the retailers and outside advertiser's bid as well as the manufacturer's choice of participation rate given the positions of all advertisers. Then we compare the manufacturer's profits among six all position configurations to identify the equilibrium.

- In the first case, retailer 1 and 2 take position 1 and 2 respectively. This happens when,

$$\frac{e_1\theta_1r_1}{1 - \alpha_1} \geq \frac{e_2\theta_2r_2}{1 - \alpha_2} \geq e_Av_A. \quad (\text{xii})$$

The manufacturer's profit is,

$$\pi_M(\alpha_1, \alpha_2) = x_1e_1 \left[\theta_1m_1 - \alpha_1 \frac{e_2}{e_1} \left(\frac{x_1 - x_2}{x_1} \frac{\theta_2r_2}{1 - \alpha_2} + \frac{x_2}{x_1} \frac{e_Av_A}{e_2} \right) \right] + x_2e_2 \left(\theta_2m_2 - \alpha_2 \frac{e_A}{e_2}v_A \right),$$

which decreases in α_1, α_2 . Therefore the manufacturer will choose the smallest α_1, α_2 that satisfy (xii). The optimal participation rates are,

$$\alpha_i^* = 1 - \frac{e_i\theta_i r_i}{e_Av_A}, \quad i = 1, 2.$$

Correspondingly, the manufacturer's maximum profit is,

$$\pi_M(\alpha_1^*, \alpha_2^*) = x_1[e_1\theta_1(m_1 + r_1) - e_Av_A] + x_2[e_2\theta_2(m_2 + r_2) - e_Av_A].$$

- In the second case, retailer 1 and 2 take position 1 and 3 respectively. This happens when,

$$\frac{e_1\theta_1r_1}{1 - \alpha_1} \geq e_Av_A \geq \frac{e_2\theta_2r_2}{1 - \alpha_2}. \quad (\text{xiii})$$

The manufacturer's profit is,

$$\pi_M(\alpha_1, \alpha_2) = x_1e_1 \left[\theta_1m_1 - \alpha_1 \frac{e_A}{e_1} \left(\frac{x_1 - x_2}{x_1} v_A + \frac{x_2}{x_1} \frac{e_2}{e_A} \frac{\theta_2r_2}{1 - \alpha_2} \right) \right],$$

which decreases in α_1, α_2 . Therefore the manufacturer will choose the smallest α_1, α_2 that satisfy (xiii). The optimal participation rates are,

$$\begin{aligned}\alpha_1^* &= 1 - \frac{e_1\theta_1r_1}{e_Av_A}, \\ \alpha_2^* &= 0.\end{aligned}$$

Correspondingly, the manufacturer's profit is

$$\pi_M(\alpha_1^*, \alpha_2^*) = x_1 [e_1\theta_1(m_1 + r_1) - e_Av_A] + x_2 \left[e_Av_A - e_1\theta_1r_1 - e_2\theta_2r_2 + \frac{e_1\theta_1r_1 \cdot e_2\theta_2r_2}{e_Av_A} \right].$$

- In the third case, retailer 1 and 2 take position 2 and 3 respectively. This happens when,

$$e_Av_A \geq \frac{e_1\theta_1r_1}{1 - \alpha_1} \geq \frac{e_2\theta_2r_2}{1 - \alpha_2}. \quad (\text{xiv})$$

The manufacturer's profit is,

$$\pi_M(\alpha_1, \alpha_2) = x_2e_1 \left[\theta_1m_1 - \alpha_1 \frac{e_2}{e_1} \frac{\theta_2r_2}{1 - \alpha_2} \right],$$

which decreases in α_1, α_2 . Therefore the manufacturer will choose the smallest α_1, α_2 that satisfy (xiv). The optimal participation rates are,

$$\begin{aligned}\alpha_1^* &= \frac{\max\{e_1\theta_1r_1, e_2\theta_2r_2\} - e_1\theta_1r_1}{e_2\theta_2r_2}, \\ \alpha_2^* &= 0.\end{aligned}$$

Correspondingly, the manufacturer's profit is

$$\pi_M(\alpha_1^*, \alpha_2^*) = x_2 [e_1\theta_1(m_1 + r_1) - \max\{e_1\theta_1r_1, e_2\theta_2r_2\}].$$

The other three cases can be obtained by symmetry. The remaining analysis is straightforward and the same with 2R model, thus omitted.

REFERENCES

- Susan Athey and Glenn Ellison. Position auctions with consumer search. *The Quarterly Journal of Economics*, page qjr028, 2011.
- Mark Bergen and George John. Understanding cooperative advertising participation rates in conventional channels. *Journal of Marketing Research*, pages 357–369, 1997.
- Paul D Berger. Vertical cooperative advertising ventures. *Journal of Marketing Research*, pages 309–312, 1972.
- Giacomo Bonanno and John Vickers. Vertical separation. *The Journal of Industrial Economics*, pages 257–265, 1988.
- Yongmin Chen and Chuan He. Paid placement: Advertising and search on the internet*. *The Economic Journal*, 121(556):F309–F328, 2011.
- Anne T Coughlan and Birger Wernerfelt. On credible delegation by oligopolists: A discussion of distribution channel management. *Management Science*, 35(2):226–239, 1989.
- Chrysanthos Dellarocas. Double marginalization in performance-based advertising: Implications and solutions. *Management Science*, 58(6):1178–1195, 2012.
- Preyas Desai. Franchise pricing strategies. *doctoral dis*, 1992.
- Preyas S Desai, Woochoel Shin, and Richard Staelin. The company that you keep: When to buy a competitor’s keyword. *Marketing Science*, 33(4):485–508, 2014.
- Benjamin Edelman, Michael Ostrovsky, and Michael Schwarz. Internet advertising and the generalized second-price auction: Selling billions of dollars worth of keywords. *American Economic Review*, 97(1):242–259, 2007.
- Juan Feng. Optimal mechanism for selling a set of commonly ranked objects. *Marketing Science*, 27(3):501–512, 2008.
- Ganesh Iyer. Coordinating channels under price and nonprice competition. *Marketing Science*, 17(4):338–355, 1998.
- Kinshuk Jerath, Liye Ma, Young-Hoon Park, and Kannan Srinivasan. A position paradox in sponsored search auctions. *Marketing Science*, 30(4):612–627, 2011.

- Abel P Jeuland and Steven M Shugan. Managing channel profits. *Marketing science*, 2(3):239–272, 1983.
- Zsolt Katona and Miklos Sarvary. The race for sponsored links: Bidding patterns for search advertising. *Marketing Science*, 29(2):199–215, 2010.
- Sang Yong Kim and Richard Staelin. Manufacturer allowances and retailer pass-through rates in a competitive environment. *Marketing Science*, 18(1):59–76, 1999.
- Shijie Lu, Yi Zhu, and Anthony Dukes. Position auctions with budget constraints: Implications for advertisers and publishers. *Marketing Science*, 34(6):897–905, 2015.
- Timothy W McGuire and Richard Staelin. An industry equilibrium analysis of downstream vertical integration. *Marketing science*, 2(2):161–191, 1983.
- K Sridhar Moorthy. Comment-managing channel profits: Comment. *Marketing Science*, 6(4):375–379, 1987.
- Sridhar Narayanan and Kirthi Kalyanam. Position effects in search advertising and their moderators: A regression discontinuity approach. *Marketing Science*, 34(3):388–407, 2015.
- Amin Sayedi, Kinshuk Jerath, and Kannan Srinivasan. Competitive poaching in sponsored search advertising and its strategic impact on traditional advertising. *Marketing Science*, 33(4):586–608, 2014.
- Hal R Varian. Position auctions. *international Journal of industrial Organization*, 25(6):1163–1178, 2007.
- J Miguel Villas-Boas. Product line design for a distribution channel. *Marketing Science*, 17(2):156–169, 1998.
- Kenneth C Wilbur and Yi Zhu. Click fraud. *Marketing Science*, 28(2):293–308, 2009.
- Ralph A Winter. Vertical control and price versus nonprice competition. *The Quarterly Journal of Economics*, pages 61–76, 1993.
- Lizhen Xu, Jianqing Chen, and Andrew Whinston. Price competition and endogenous valuation in search advertising. *Journal of Marketing Research*, 48(3):566–586, 2011.
- Yi Zhu and Kenneth C Wilbur. Hybrid advertising auctions. *Marketing Science*, 30(2):249–273, 2011.